 DOI:10.30420/565822236

Mathematical Model of the Temperature Rise of a Wireless Power Transfer Coil in DC Operation

Daniel Gückelhorn[2], Christian Merz[1], Cem Som[1]

[1] Würth Elektronik eiSos GmbH & Co. KG, Germany
[2] Munich University of Applied Sciences, Germany

Corresponding author: Christian Merz, christian.merz@we-online.de

Abstract

In this paper, a mathematical model, which is used for calculating the temperature rise of a wireless power transfer coil in dependency of the DC current, is introduced. This simplified and time efficient model is implemented into a Python script allowing an estimation of the rated current of the wireless power transfer coil resulting in faster prototyping and cost savings. For the mathematical description, an equivalent electrical model was chosen for the well-known dimensions and material properties of such a coil. The derivation of the needed differential equations is described in detail. To verify the model, the calculated heating curves are compared with measurements.

1 Introduction

For wireless power transfer (WPT) coils the so-called "rated current" is an important parameter to operate the component securely. The rated current is defined as the DC current, which causes a temperature rise of 40 °C in the steady state at the surface of the coil. The measurement is very time consuming and susceptible to errors during the process. Imprecisely defined parameters like the placement of the temperature sensor or undefined parameters like the convection of air can cause large variations of the rated current as described in [1]. Another possibility is to use a simulation tool. These simulation tools usually have high costs and require an experienced user. To determine the rated current for a first prototype design both options are not suitable. A more efficient solution is an easy-to-use Python script. This script allows calculating the rated current curve of a temperature rise between 0 °C and 60 °C in a few seconds with a minimal of user inputs. In this paper, the required physical and mathematical models are described. For simplicity the models are only valid for round coils, which is the most common shape. The symmetry allows faster implementation of the equations. Further shapes can also be included with little effort.

2 Methods and Models

2.1 Thermal Model

The model of the WPT coil consists of four parts: the copper wire, the isolation layer, the epoxy layer, and the ferrite plate, as shown in Fig. 1. To

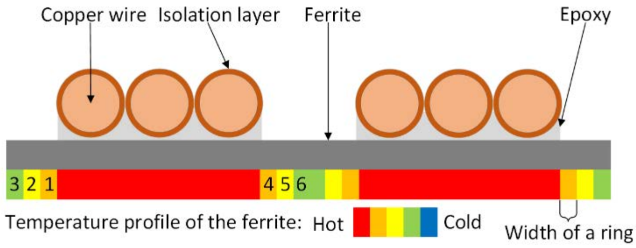

Fig. 1 Cross section of the used physical model of a WPT coil with the temperature distribution of the ferrite rings.

achieve a model which is easy to implement and can be calculated quickly two assumptions have to be made. First, the temperature of the copper wire, the isolation layer and the epoxy have a homogeneous temperature profile. That results in a very short and easy calculation of the temperature of these parts, because only one value must be determined. The other assumption is that there is no heat transfer between the backside of the ferrite and the environment. This is

ISBN 978-3-8007-5822-7

done because in the worst case the WPT coil is fixed on a plastic case with bad heat conductivity. For the ferrite plate, a homogeneous temperature profile is not realistic for the most coils due to the size of the plate. To take this into account the ferrite plate is separated into rings where each ring has a uniform temperature. The area of the ferrite that is located directly under the coil is considered as one large ring with the highest temperature. The remaining area is separated into rings with an adjustable width (2 to 6). The model uses a default width of 1 mm. Due to the symmetry, it is enough to consider one-half of the cross section. To calculate temperatures of round coils with a square shaped ferrite, the ferrite is converted into a round shape with the same surface area.

2.2 Mathematical Model

For the calculation of the temperature rise of the coil an equivalent electrical circuit is used. This is done, because the exact dimensions and material properties are known, so the resulting thermal parameters can be calculated precisely. The equivalent electrical circuit allows to represent the thermal model as an electrical circuit with the well-known components and mathematical sets. The components used in a thermal model and in a real electrical circuit differ by notation and units. However, the meaning is the same. The thermal and their electrical equivalent components are shown in table 1. Detailed explanations about the setup of a thermal system can be found in [2]. The copper wire with its electrical resistance is modeled as a heat source. The heat

Thermal Model	Electrical Circuit
Q_t Heat	q Charge
q_t Heat Flow Rate	i Current
---	L Inductance
C_t Thermal Capacitance	C Capacitance
R_t Thermal Resistance	R Resistance
T Temperature	V Voltage

Table 1 Comparison of the quantities of a thermal model and the resulting quantities in an equivalent electrical circuit.

flows from the copper wire to the isolation layer. The contact resistance between two different solids is neglected, only the thermal resistance of the materials is considered. From the isolation layer the heat flux is divided between the epoxy layer and the environment because of radiation and convection. Thermal flux caused by conduction through the air is neglected because of the bad heat conductivity of air. Now, the heat flows through the epoxy layer into the area of the ferrite sitting directly under the coil. A part of the heat flux flows to the outside and the other part flows to the center of the ferrite. With each ring the heat flux passes, thermal radiation and convection dissipate a part of the heat. The model is shown in Fig. 2. In total there are N outer ferrite rings and i inner ferrite rings. The exact number depends on the ferrite size. That leads to 4 + N + i temperature values.

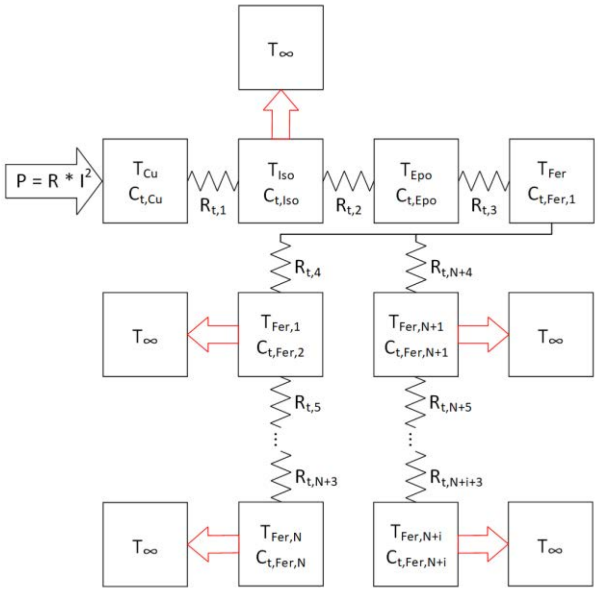

Fig. 2 Equivalent electrical circuit of the physical WPT coil model. The blocks $T_{Fer,1}$ to $T_{Fer,N+i}$ represent ferrite rings. The red arrows indicate the heat dissipation caused by thermal radiation and convection. T_∞ is the ambient temperature.

3 Calculation

3.1 Setup of the Equations

For every part and ferrite ring of the WPT coil the temperature can be calculated with the heat balance equation:

$$C_t \frac{dT}{dt} = \sum_{i=0}^{n} q_t \quad (1)$$

The thermal capacitance can be calculated from the known volume V, mass density ρ and specific heat capacity c with Eq. (2). The sum of the heat flux rate is equivalent to Kirchhoff's current law. Incoming heat flux is assumed to be positive and

$$C_t = \rho V c \quad (2)$$

outgoing heat flux is supposed to be negative. In the model there are three different kinds of heat flux:

1. Heat flux between solid 1 and solid 2:

$$q_t = \frac{T_1 - T_2}{R_t} \quad (3)$$

2. Thermal convection:

$$q_t = \alpha A (T - T_\infty) \quad (4)$$

With the heat transfer coefficient α and the surface area A.

3. Thermal radiation:

$$q_t = \varepsilon \sigma A (T^4 - T_\infty^4) \quad (5)$$

With the emissivity ε and the Stefan-Boltzmann constant σ.

The thermal resistance can be determined with,

$$R_t = \frac{d}{\lambda \mathrm{A}} \quad (6)$$

where d is the thickness and λ is the heat conductivity of the material.

The resulting equations for the different parts of the coil:

1. Copper wire:

$$C_{Cu} \frac{dT_{Cu}}{dt} = P - \frac{T_{Cu} - T_{Iso}}{R_{t,1}} \quad (7)$$

2. Isolation layer:

$$C_{Iso} \frac{dT_{Iso}}{dt} = \frac{T_{Cu} - T_{Iso}}{R_{t,1}} - \frac{T_{Iso} - T_{Epo}}{R_{t,2}} - \alpha A_1 (T_{Iso} - T_\infty) - \sigma A_1 (T_{Iso}^4 - T_\infty^4) \quad (8)$$

3. Epoxy:

$$C_{Epo} \frac{dT_{Epo}}{dt} = \frac{T_{Iso} - T_{Epo}}{R_{t,2}} - \frac{T_{Epo} - T_{Fer}}{R_{t,3}} \quad (9)$$

4. Ferrite:

$$C_{Fer} \frac{dT_{Fer}}{dt} = \frac{T_{Epo} - T_{Fer}}{R_{t,3}} - \frac{T_{Fer} - T_{Fer,1}}{R_{t,4}} - \frac{T_{Fer} - T_{Fer,N+1}}{R_{t,N+4}} \quad (10)$$

5. Ferrite ring 1:

$$C_{Fer,1} \frac{dT_{Fer,1}}{dt} = \frac{T_{Fer} - T_{Fer,1}}{R_{t,4}} - \frac{T_{Fer,1} - T_{Fer,2}}{R_{t,5}} - \alpha A_2 (T_{Fer,1} - T_\infty) - \sigma A_2 (T_{Fer,1}^4 - T_\infty^4) \quad (11)$$

6. Ferrite ring N:

$$C_{Fer,N} \frac{dT_{Fer,N}}{dt} = \frac{T_{Fer,N-1} - T_{Fer,N}}{R_{t,N+3}} - \alpha A_{N+1} (T_{Fer,N} - T_\infty) - \sigma A_{N+1} (T_{Fer,N}^4 - T_\infty^4) \quad (12)$$

Before this set of coupled differential equations can be solved the thermal capacities, thermal resistances, surfaces areas for thermal convection and radiation, and the heat transfer coefficient must be determined.

3.2 Required Input Parameters

To calculate the necessary parameters some user inputs of the geometrical properties of the coil are

User input	Note
Number of parallel wires	Unifilar, bifilar etc.
Wire type	Solid or litz wire
Wire diameter	
Litz strain diameter	If litz wire is used
Number of litz strains	If litz wire is used
Outer coil radius	
Number of turns	Turns per layer
Number of layers	
Ferrite shape	Round or square
Inner ferrite radius	If ferrite has a hole
Outer ferrite radius	If ferrite is round
Side length of ferrite	If ferrite is square
Ferrite thickness	

Table 2 Required user inputs of the coil geometry to calculate the thermal parameters.

Material	c (J/(kgK))	ρ (kg/m³)	λ (W/(mK))
Copper	385	8960	---
Isolation	440	1200	0.4
Epoxy	1000	1200	0.2
Ferrite	700	5000	5.0

Table 3 Used material parameters.

required. These user inputs, shown in table 2, are requested by the used Python script to perform the calculations. In addition, material parameters are needed, which are shown in table 3.

3.3 Determination of the Thermal Capacities

For the calculation Eq. (2) is used. The specific heat capacity and mass density are known material parameters, so only the volumes have to be determined. The thermal capacities of the individual parts are:

1. C_{Cu}:

The wire volume can be calculated with,

$$V_{Cu} = \pi \frac{d_{Wire}^2}{4} l_{Wire} \tag{13}$$

where d_{Wire} is the wire diameter and l_{Wire} is the wire length. The wire length can be determined with the mean radius of the coil,

$$l_{Wire} = \pi(r_{a,Coil} + r_{i,Coil}) n_{Turn} n_{Layer} \tag{14}$$

with the outer coil radius $r_{a,Coil}$, the inner coil radius $r_{i,Coil}$, the number of turns n_{Turn} and the number of layers n_{Layer}. The inner coil radius can be calculated with Eq. (15). n_P is the amount of parallel wires.

$$r_{i,Coil} = r_{a,Coil} - n_{Turn} d_{Wire} n_P \tag{15}$$

The resulting thermal capacity is:

$$C_{Cu} = \rho_{Cu} V_{Cu} c_{Cu} \tag{16}$$

2. C_{Iso}:

The volume of the isolation can be approximated with Eq. (17). t_{Iso} is the thickness of the isolation layer. This quantity is usually in the range of micrometers. In this case 40 µm are assumed. Since the layer is very thin in comparison to the wire diameter a value in an appropriate range is sufficient. The thermal capacity is calculated with Eq. (18).

$$V_{Iso} = t_{Iso} \pi d_{Wire} l_{Wire} \tag{17}$$

$$C_{Iso} = \rho_{Iso} V_{Iso} c_{Iso} \tag{18}$$

3. C_{Epo}:

The volume of the epoxy has to be estimated. It is assumed that the epoxy is filling the cavities between windings and layers. That leads to Eq. (19) and Eq. (20).

$$V_{Epo} = \left((\pi r_{a,Coil}^2 - \pi r_{i,Coil}^2) \frac{d_{Wire}}{2} - \pi \frac{d_{Wire}^2}{8} l_{Wire}\right)\left(n_{Layer} - \frac{1}{2}\right) \tag{19}$$

$$C_{Epo} = \rho_{Epo} V_{Epo} c_{Epo} \tag{20}$$

4. C_{Fer}:

For the ferrite directly under the coil, the volume is,

$$V_{Fer} = (\pi r_{a,Coil}^2 - \pi r_{i,Coil}^2) t_{Fer} \tag{21}$$

with the ferrite thickness t_{Fer}. The resulting thermal capacity is:

$$C_{Fer} = \rho_{Fer} V_{Fer} c_{Fer} \tag{22}$$

5. $C_{Fer,1}$:

The volume of the first outer ferrite ring can be calculated with,

$$V_{Fer,1} = (\pi(w_{Ring} + r_{a,Coil})^2 - \pi r_{a,Coil}^2) t_{Fer} \tag{23}$$

where w_{Ring} is the width of a ferrite ring. The heat capacity is calculated with Eq. (24). Other outer ferrite rings follow the same calculation scheme.

$$C_{Fer,1} = \rho_{Fer,1} V_{Fer,1} c_{Fer,1} \tag{24}$$

6. $C_{Fer,N+1}$:

The equation for the first inner ferrite ring is similar to the outer ferrite ring:

$$V_{Fer,N+1} = (\pi {r_{i,Coil}}^2 - \pi(r_{i,Coil} - w_{Ring})^2) t_{Fer} \tag{25}$$

The heat capacity is:

$$C_{Fer,N+1} = \rho_{Fer,N+1} V_{Fer,N+1} c_{Fer,N+1} \tag{26}$$

The remaining inner ferrite rings can be calculated in the same way.

3.4 Determination of the Thermal Resistances

The thermal resistances are calculated with Eq. (6):

1. $R_{t,1}$:

$$R_{t,1} = \frac{t_{Iso}}{\lambda_{Iso} \pi d_{Wire} l_{Wire}} \tag{27}$$

2. $R_{t,2}$:

For the calculation of the thermal resistance for the epoxy it is assumed that only the epoxy in the cavities between the first layer and the ferrite contributes to the thermal resistance. The resulting thermal resistance is calculated with Eq. (26).

$$R_{t,2} = \frac{t_{Epo}}{\lambda_{Epo}\pi \frac{d_{Wire}}{2} l_{Wire}} \quad (28)$$

The thickness of the epoxy layer t_{Epo} is:

$$t_{Epo} = \frac{\pi\left(\left(r_{a,Coil}^2 - r_{i,Coil}^2\right) - \left(\frac{d_{Wire}}{4} l_{Wire}\right)\right)}{\pi l_{Wire}} \quad (29)$$

3. $R_{t,3}$:

$$R_{t,3} = \frac{t_{Fer}}{\lambda_{Fer}\left(\pi r_{a,Coil}^2 - \pi r_{i,Coil}^2\right)} \quad (30)$$

4. $R_{t,4}$:

$$R_{t,4} = \frac{w_{ring}}{\lambda_{Fer} t_{Fer} 2\pi r_{a,Coil}} \quad (31)$$

The thermal resistance for the other outer ferrite rings can be calculated with the same procedure.

5. $R_{t,N+4}$:

$$R_{t,N+4} = \frac{w_{ring}}{\lambda_{Fer} t_{Fer} 2\pi r_{i,Coil}} \quad (32)$$

For the remaining inner ferrite rings the calculation follows the same scheme.

3.5 Determination of the Surface Areas

The surface areas for thermal convection and radiation can be calculated as follow:

1. A_1:

For the surface area only the outer windings contacting the environment are considered. With a one-layer coil half of the windings are covered with epoxy, so this area is not contributing to the environmental exchange. With every additional layer a surface area equivalent of a single winding is added. The equation is:

$$A_1 = \frac{\pi^2\left(r_{a,Coil} + r_{i,Coil}\right) d_{Wire} n_{Turn}}{2} + \pi^2\left(r_{a,Coil} + r_{i,Coil}\right) d_{Wire}\left(n_{Layer} - 1\right) \quad (33)$$

2. A_2:

The first outer ferrite ring has the area:

$$A_2 = \pi\left(w_{Ring} + r_{a,Coil}\right)^2 - \pi r_{a,Coil}^2 \quad (34)$$

The surface area of the other outer rings can be determined similarly.

3. A_{N+2}:

The surface area of the first inner ferrite ring is:

$$V_{Fer,N+1} = \pi {r_{i,Coil}}^2 - \pi\left(r_{i,Coil} - w_{Ring}\right)^2 \quad (35)$$

The remaining inner ferrite surface areas follow are calculated in the same way.

3.6 Determination of the Heat Transfer Coefficient

The heat transfer coefficient depends on many system parameters so there are no standardized values for different materials. However, there are approximations for different cases. It can be assumed that the copper wire and ferrite are metal walls. For metal walls the heat transfer coefficient is between 3.5 W/(m^{2}K) and 35 W/(m^{2}K) [3]. To get a more accurate value, the rated current curve is calculated for different heat transfer coefficients and compared to measurements. The value with the lowest deviation between calculation and measurement was chosen. This was done for six WPT coils from Würth Elektronik eiSos GmbH & Co. KG (WE). The tested coils, the optimal heat transfer coefficients and resulting relative deviation are shown in table 4. For simplicity the heat transfer coefficient value was limited to integers.

Product number	α (W/(m^{2}K))	ΔI_R (%)
760308101410	11	1.20
760308100143	11	0.96
760308102142	14	0.93
760308101214	16	0.12
760308101309	26	-0.68
760308102212	22	-0.49

Table 4 Overview of the used WPT coils with their optimal heat transfer coefficient values and the resulting relative deviation of the rated current.

The coils 760308101410, 760308100143 and 760308102142 have a diameter of around 50 mm and are made of litz wire. The other three coils have a diameter between 20 mm and 30 mm and are made from solid wire. That is why these coils are split up in two groups. For every group the mean value of the heat transfer coefficient is calculated. For the larger coils the value is 12 W/(m^{2}K) and for the smaller coils 21.3 W/(m^{2}K). That mean value can now be used to calculate any coil having a similar shape like the coils used in the groups. With increasing number of measurements more accurate heat transfer coefficients for each coil design can be calculated. A database of measurements of different existing coil designs can be analyzed to achieve heat transfer coefficients with high accuracy for future coils.

4 Results

The results of the larger coils of the measurement and calculation are shown in Fig. 3. The relative

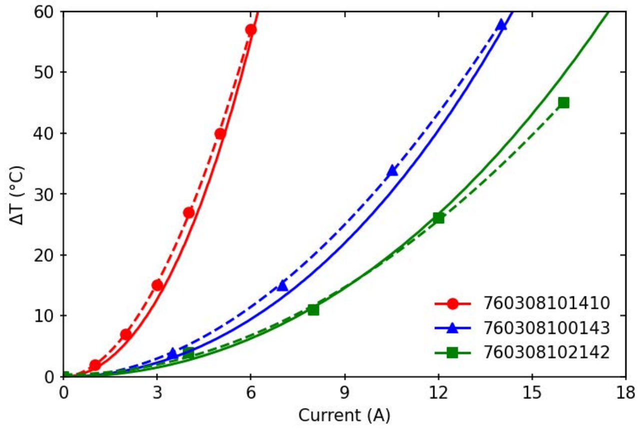

Fig. 3 Comparison of the interpolated measured rated current curves (dashed lines) and calculated rated current curve (solid lines) with α=12 W/(m^{2}K).

deviation of the rated current is below 4 % for every coil. Also, the calculated curves have a very similar shape in comparison to the measured curves. This indicates that all relevant physics is covered by the simplified models. Furthermore, the use of the averaged heat transfer coefficient delivers good results for every coil. The relative deviation of the small coils varies between 1 % and 10 %, as Fig. 4 shows. The main reason may be the greater

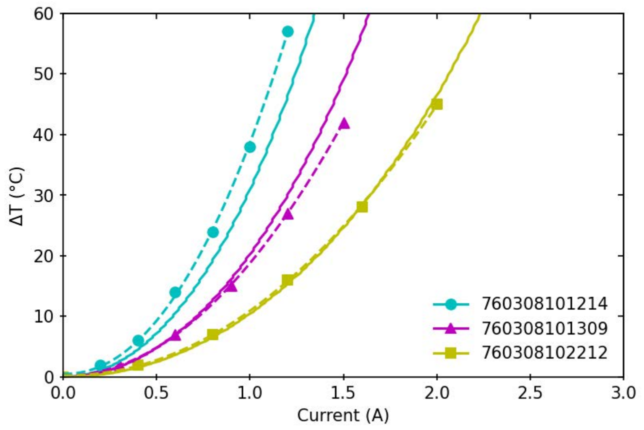

Fig. 4 Comparison of the interpolated measured rated current curves (dashed lines) and calculated rated current curve (solid lines) with α=21.3 W/(m^{2}K).

variety the heat transfer coefficients. The coil 760308101214 has the largest deviation between its heat transfer coefficient and the averaged heat transfer coefficient. Nevertheless, the curves shapes also indicate that physics has been adequately covered. These results indicate that the model is valid and can be used for further calculations. The remaining challenge is to determine reasonable heat transfer coefficients.

In addition, a Python script was written which calculates the rated current curves for every common round WPT coil with only 13 user inputs. The calculation time for each coil was less than 5 s.

5 Summary

In this paper, an easy to implement and time efficient model to calculate the rated DC current of a round WPT coil was achieved with a total calculation time of less than 5 s. The models were verified by measurements of six WPT coils from WE. The calculation is very accurate with a deviation of around 4 % for larger coils and a maximum deviation of 10 % for smaller coils. The overall challenge of this model is the precise determination of the heat transfer coefficient. In this paper this was achieved with measurements because the coefficient is dependent on many parameters and standardized values for different materials are not available.

6 Outlook

Based on the presented models WE is working on the calculation of the rated current in AC operation for different frequencies. This requires the inclusion of the skin effect and the proximity effect. Extensive work in the topic of AC behavior of WPT coils has already been done in [4].

7 References

[1] Richard Blakey, Alexander Gerfer, “Application Note: Was bedeutet Nennstrom?”, Würth Elektronik eiSos GmbH & Co. KG, 2021.

[2] Hans Dieter Baehr, Karl Stephan, „Wärme- und Stoffübertragung“, Springer, 7th edition, 2010.

[3] Horst Kuchling, “Taschenbuch der Physik“, Hanser, 2014.

[4] Christoph Utschick, Christian Merz, Cem Som, “AC Loss Behavior of Wireless Power Transfer Coils”, IEEE Wireless Power Transfer Conference (WPTC), 2019.

