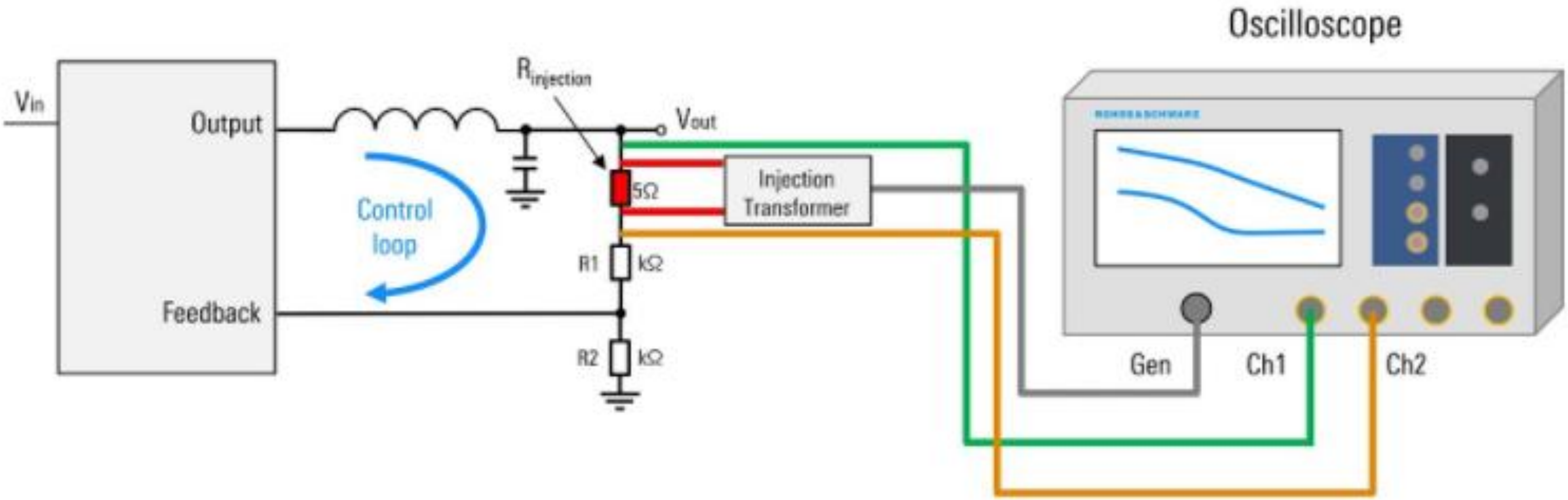
ART OF LOOP COMPENSATION

Control theory and loop

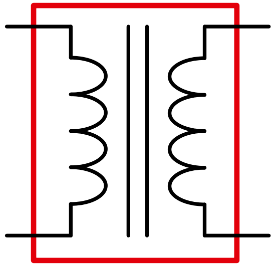
POSSIBLE SETUP

Bode plot of the control loop (plant + compensator)

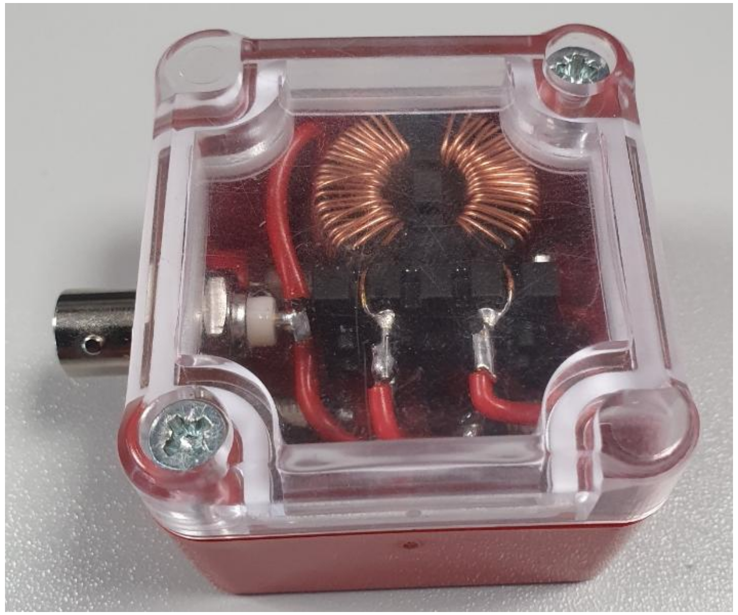


LOOP COMPENSATION

DIY Injection transformer

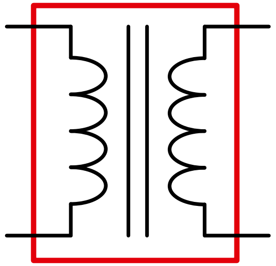


- Use a wideband common mode choke as small signal transformer
- BOM:
  - Choke: **35mH** WE-CMBNC #7448040435
  - Box: Z116PH/RD by Kradex
  - Bananas: bil-20-sw & bil-20-rt HIRSCHMANN T&M
  - Coax: 4067878 Amphenol RF



LOOP COMPENSATION

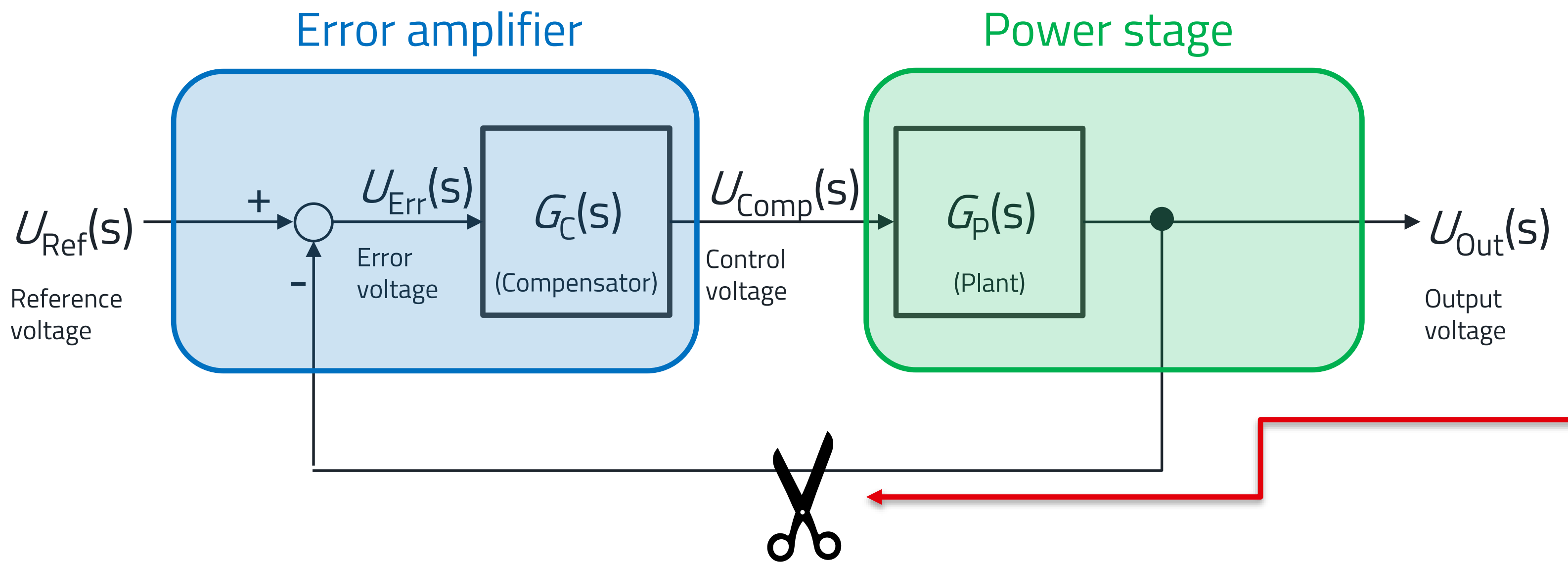
DIY Injection transformer : flat response from 10Hz to 1 MHz!



BASICS OF CONTROL THEORY

Open / Close loop transfer function

$$\text{transfer function (s)} = \frac{\text{Output (s)}}{\text{Input (s)}}$$



Closed Loop / Block-diagram

- **U_{Ref}(s):** Reference voltage = setpoint
- **U_{Err}(s):** Error voltage = Error
- **U_{Comp}(s):** Control voltage = system input
- **U_{Out}(s):** Output voltage = system output
- **G_C(s):** Transfer function of the compensator
- **G_P(s):** Transfer function of the plant
- target/actual comparison + compensator = **Error amplifier**

$$G_{OL}(s) = \frac{U_{Out}(s)}{U_{Err}(s)} = G_C(s) \cdot G_P(s)$$

➢ Open loop transfer function

$$G_{CL}(s) = \frac{U_{Out}(s)}{U_{Ref}(s)} = \frac{G_{OL}(s)}{1+G_{OL}(s)}$$

➢ Closed loop transfer function

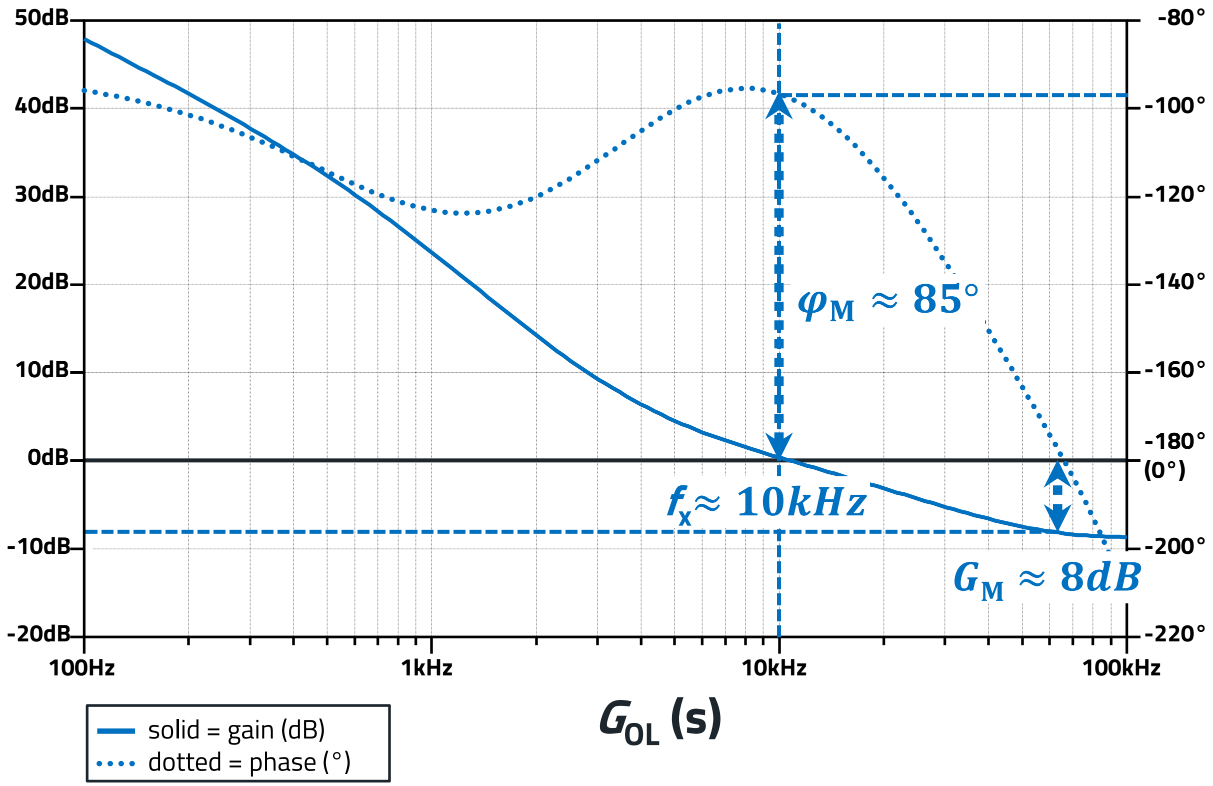
BASICS OF CONTROL THEORY

Identify stability on an open loop bode plot

- **Cross over frequency (bandwidth) - f_x**
  - Frequency at which the gain crosses 0dB („1“)
  - Usually a maximum of 1/10th of the switching frequency is desired
  - **Higher cross over frequency → Faster transient response**

- **Phase margin - φ_M**
  - Phase left to -180° when the gain reaches 0dB
  - Should be ≥ 45°(60° preferred)
  - **Lower phase margin → more oscillating in transient response (load step)**

- **Gain margin - G_M**
  - Gain below 0dB when the phase reaches -180° („-“)
  - 10-15dB is considered good
  - **Gain margin too low → low variation robustness → oscillations could be the result**

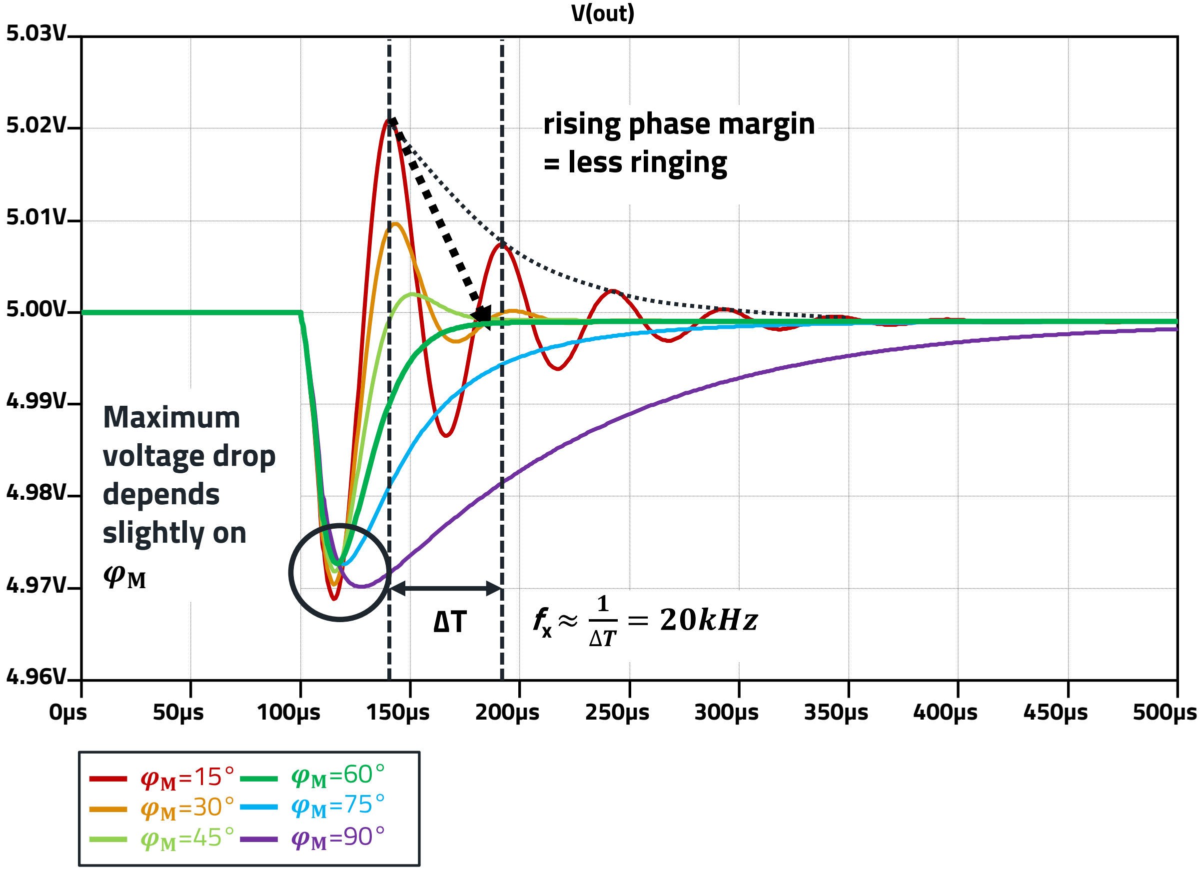


Typical bode plot – Open loop

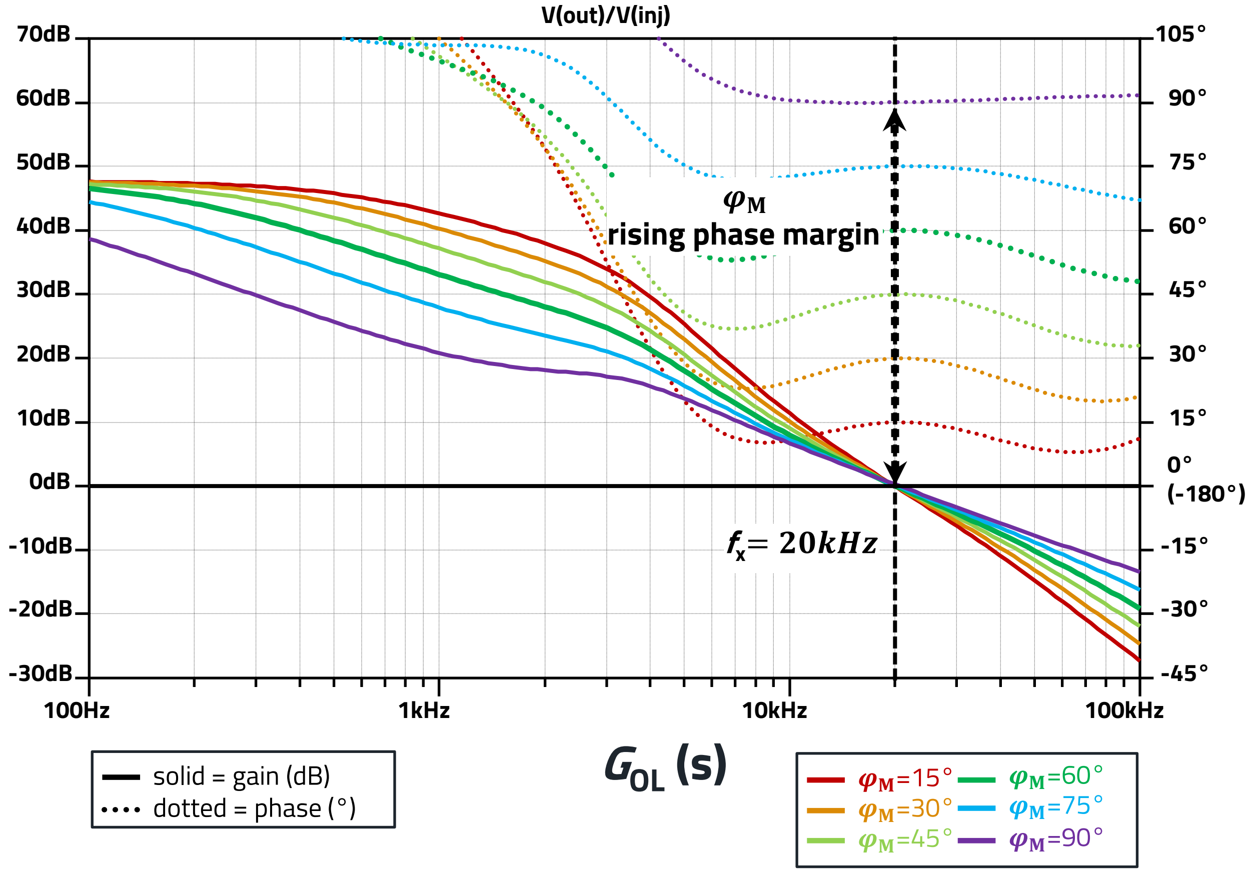
If the open loop transfer function is measured in a closed loop configuration (by Frequency Response Analysis) -180° corresponds to 0°. Reason is the inverting behavior of the error amplifier which is automatically considered. The phase margin can be read directly from 0° (this is the case in all following slides)

IMPACT OF PHASE MARGIN

Simulation – Buck Demo Board with various compensators



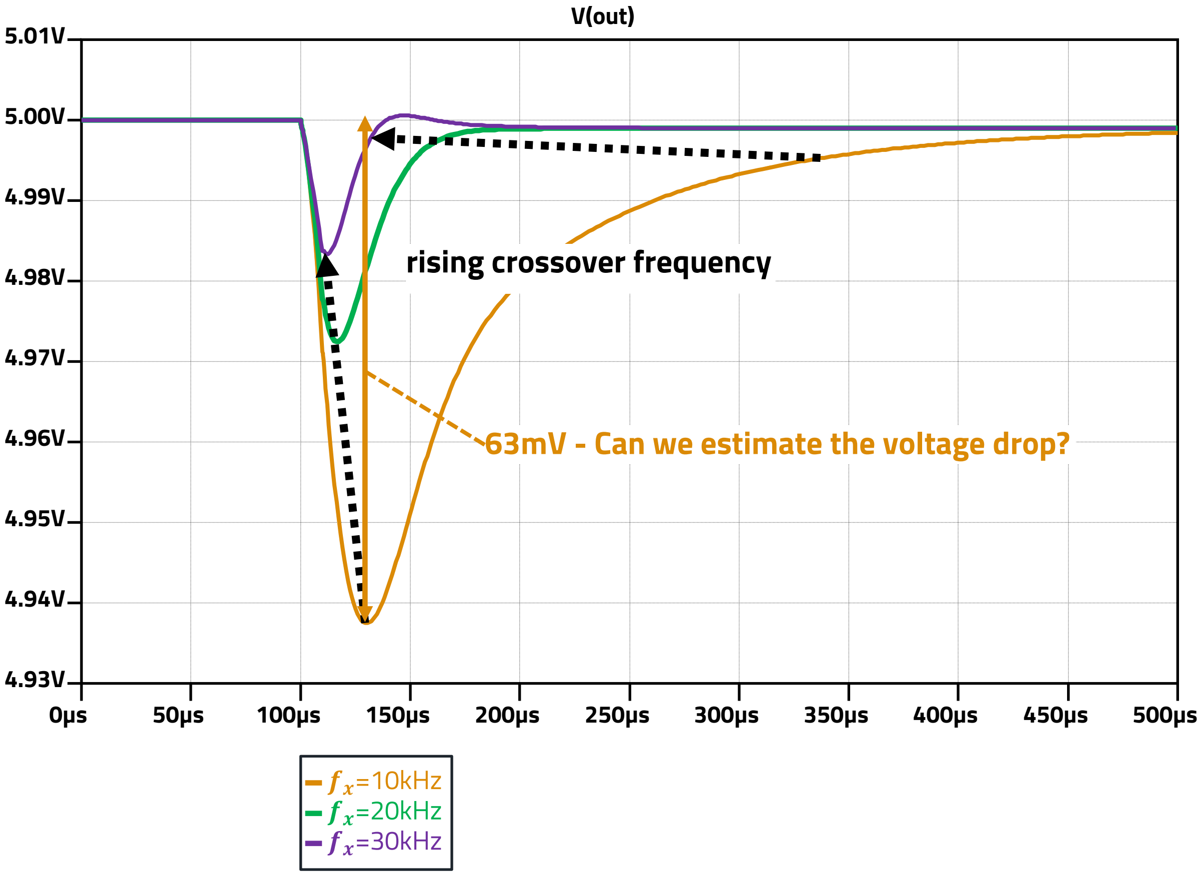
Load step response: I_{Out} 1A → 2A / U_{In}=19V



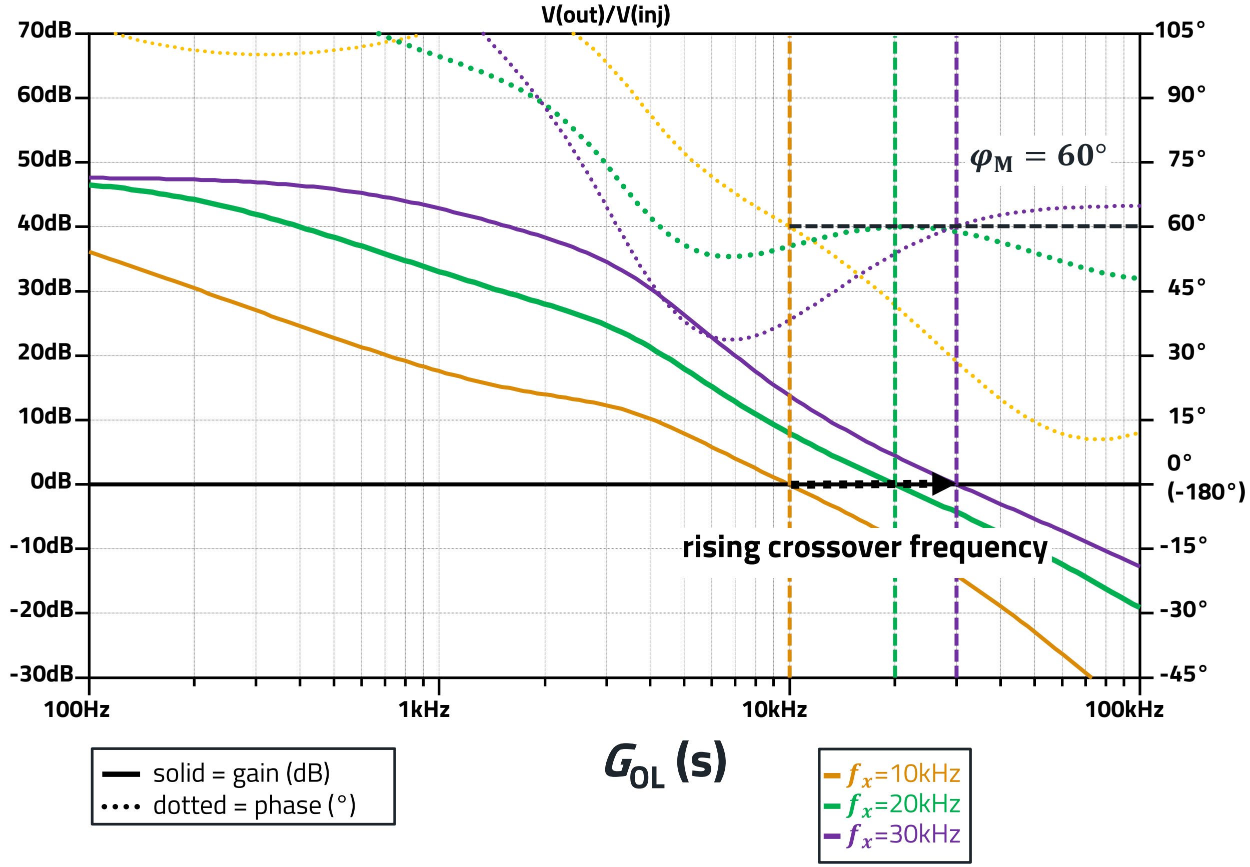
Bode plot - Open loop: I_{Out}=2A / U_{In}=19V

IMPACT OF CROSSOVER FREQUENCY

Simulation – Buck Demo Board with various compensators



Load step response: I_{Out} 1A → 2A / U_{In}=19V



Bode plot - Open loop: I_{Out}=2A / U_{In}=19V

PLANT TRANSFER FUNCTION

TRANSFER FUNCTION OF THE PLANT

What does it depend on?

- The plant transfer function depends on:
  - Control technique (e.g. voltage- / current-mode control)
  - Topology of the converter (e.g. buck, boost)
  - Conduction mode (DCM – discontinuous conduction mode, CCM – continuous conduction mode)
  - Controller IC (internal gains, compensation ramp)
  - Components used (capacitors, inductors, power semiconductors)
  - Input- and output-voltage
  - Load

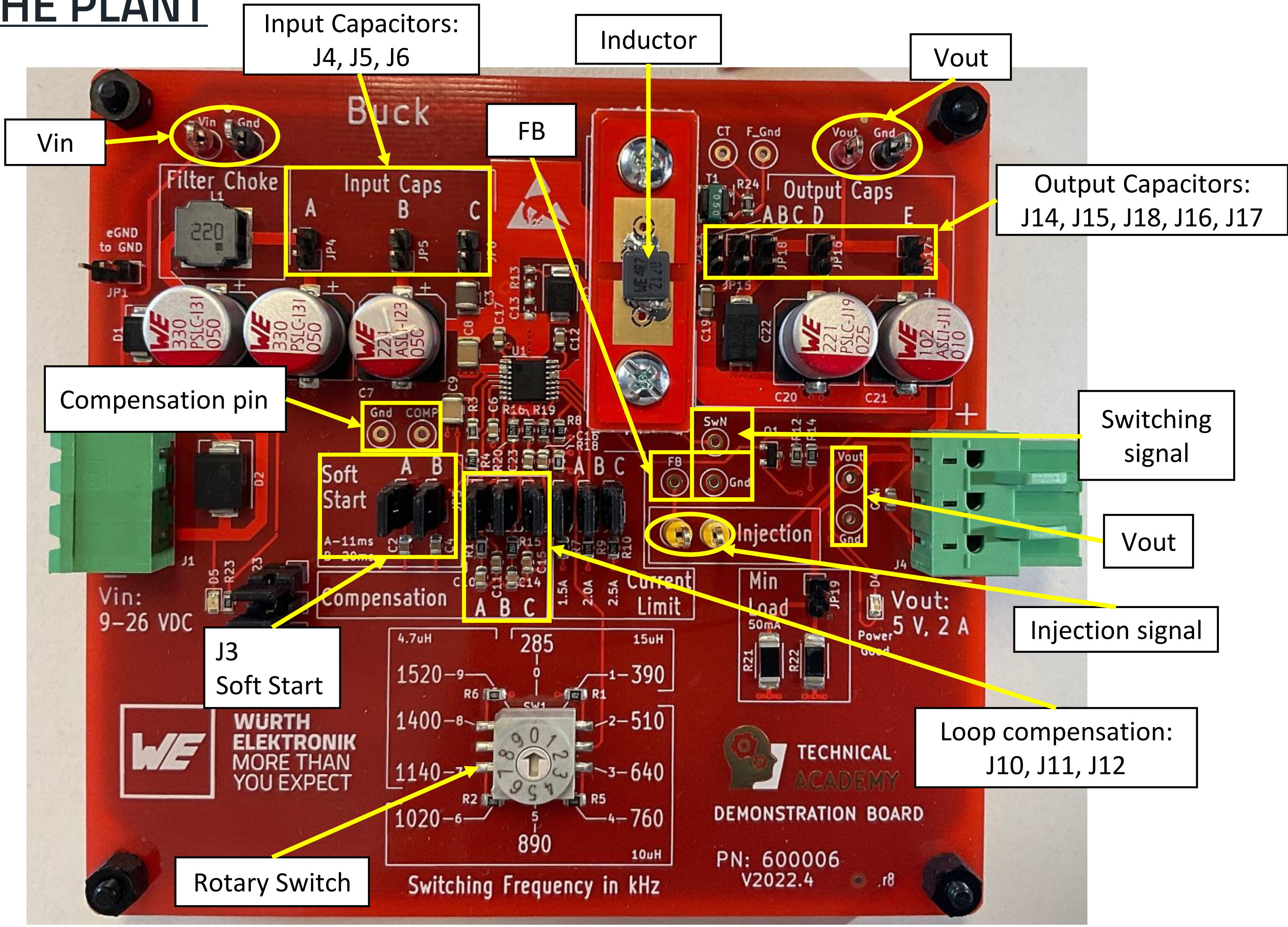
TRANSFER FUNCTION OF THE PLANT

Buck Demo Board

General Specification

- DC/DC Buck Converter
  - *Voltage Mode control*
  - *CCM*
  - V_{in} = 9-26V
  - V_{out} = 5V
  - $I_{out,max}$ = 2A
  - $P_{out,max}$ = 10W
  - f_{sw} = 285kHz – 1.52MHz

- Inductor: 10µH/3A WE-LHMI 74437346100



TRANSFER FUNCTION OF THE PLANT

Schematic and setup - Buck Demo Board

REDEXPERT

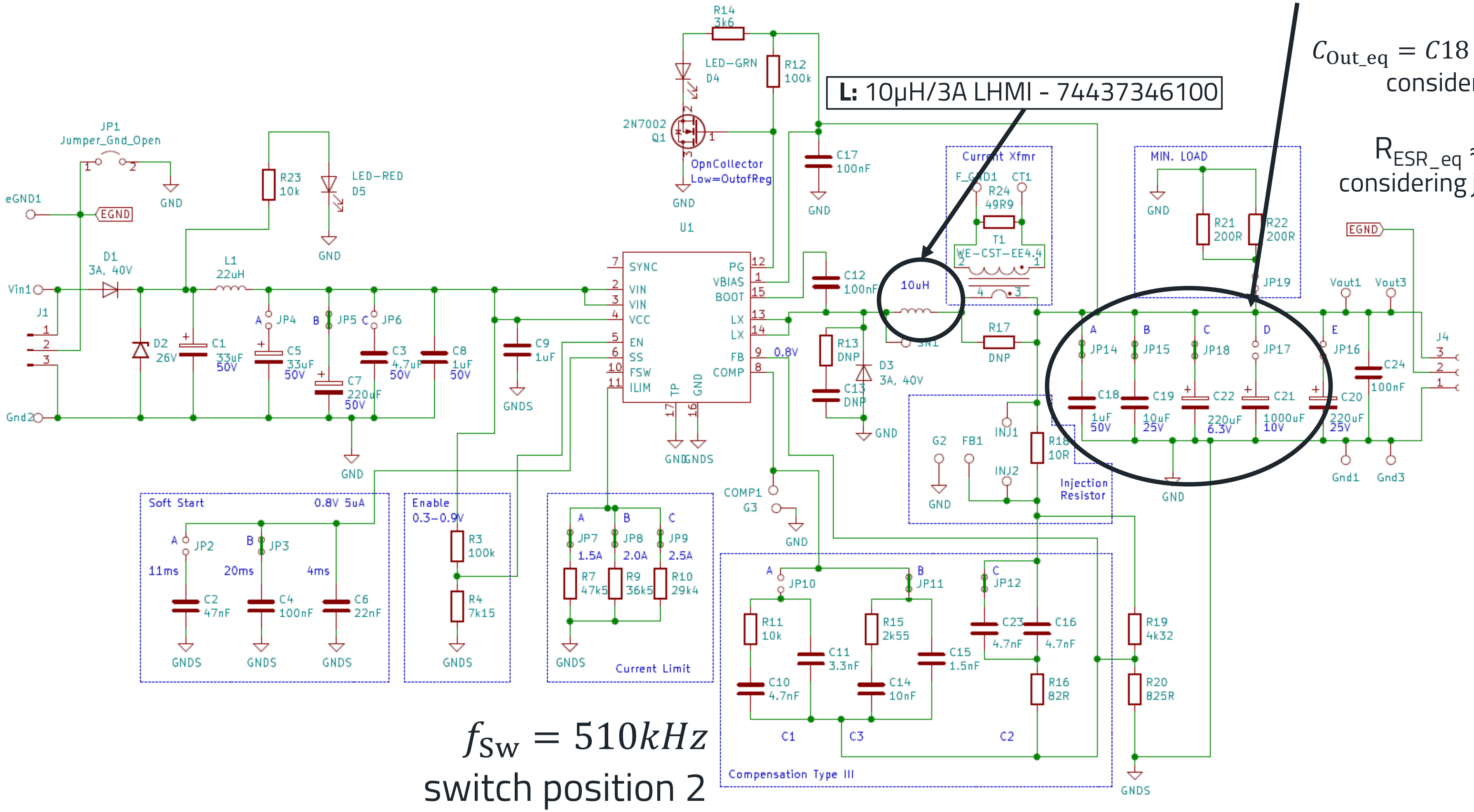
	C18	C19	C22
	885012207103 WCAP-CSGP X7R 0805 1µF 50V	**885012108021** WCAP-CSGP X5R 1206 10µF 25V	**875015119006** WCAP-PHGP H-Chip Polymer 220µF 6.3V
ESR @ 20kHz	45mΩ	22mΩ	5mΩ

$C_{Out\_eq} = C18 + C19 + C22 =$ **227.5µF**
considering DC Bias effect

$R_{ESR\_eq} \approx$ **20mΩ** at 20kHz &
considering jumper resistance, etc.

L: 10µH/3A LHMI - 74437346100

$U_{In} = 19V$

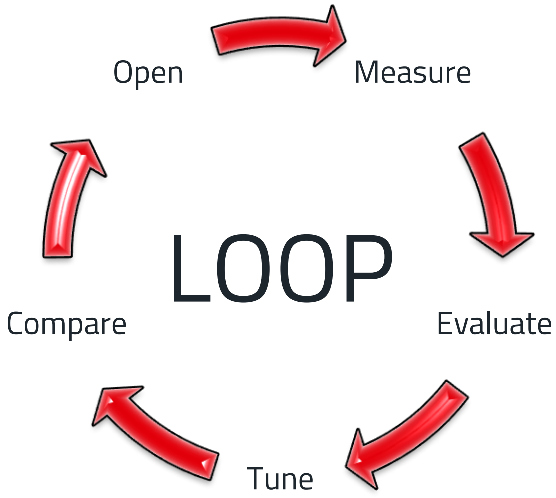


$U_{Out} = 5V$
$I_{Out} = 2A$
$R_{Load} = 2{,}5\Omega$

$f_{SW} = 510kHz$
switch position 2

TRANSFER FUNCTION OF THE PLANT

Evaluate the transfer function



- How?
  - IC **datasheet**
  - **Mathematical** modeling (MATLAB, Mathcad, etc.)
  - **Simulation** (average model of the plant)
  - **Measurements**

TRANSFER FUNCTION OF THE PLANT

Plant transfer function - model accuracy

$$G_P(s) = A_{PWM} \cdot \frac{1 + \frac{s}{\omega_{Z,ESR}}}{1 + \frac{s}{Q_0 \cdot \omega_0} + \frac{s^2}{{\omega_0}^2}}$$

- **simplified transfer function**
  - Plant (in CCM) is basically a second-order response (like a LC-filter)
  - LC double pole (complex conjugate pole)

➢ $\omega_0 = \frac{1}{\sqrt{L \cdot C_{Out}}} \rightarrow f_0 = \frac{1}{2\pi \cdot \sqrt{L \cdot C_{Out}}} \rightarrow f_0 = \frac{1}{2\pi \cdot \sqrt{10\mu H \cdot 227{,}5\mu F}} \approx \mathbf{3{,}338 kHz}$

  - Quality of the LC double pole:

➢ $Q_0 = \frac{R_{Load}}{\sqrt{\frac{L}{C_{Out}}}} = \frac{2{,}5\Omega}{\sqrt{\frac{10\mu H}{227.5\mu F}}} \approx \mathbf{11.92}$

- **more detailed transfer function**
  - LC double pole (complex conjugate pole)

➢ $\omega_0 = \frac{1}{\sqrt{L \cdot C_{Out} \cdot \left(1 + \frac{R_{ESR}}{R_{Load}}\right)}} = \frac{1}{\sqrt{10\mu H \cdot 227{,}5\mu F \cdot \left(1 + \frac{20m\Omega}{2{,}5\Omega}\right)}} \approx 20{,}882 \cdot 10^3 \frac{rad}{s}$

$\rightarrow f_0 \approx \mathbf{3{,}325 kHz}$

  - **ESR-zero:**

➢ $\omega_{Z,ESR} = \frac{1}{R_{ESR} \cdot C_{Out}} \rightarrow f_{Z,ESR} = \frac{1}{2\pi \cdot R_{ESR} \cdot C_{Out}}$

➢ $f_{Z,ESR} = \frac{1}{2\pi \cdot 20m\Omega \cdot 227{,}5\mu F} \approx \mathbf{34{,}997 kHz}$

➢ $OR\ f_{Z,ESR} = \frac{1}{2\pi \cdot 10m\Omega \cdot 231\mu F} \approx \mathbf{68{,}9 kHz}$ **??**

  - **Quality of the LC double pole:**

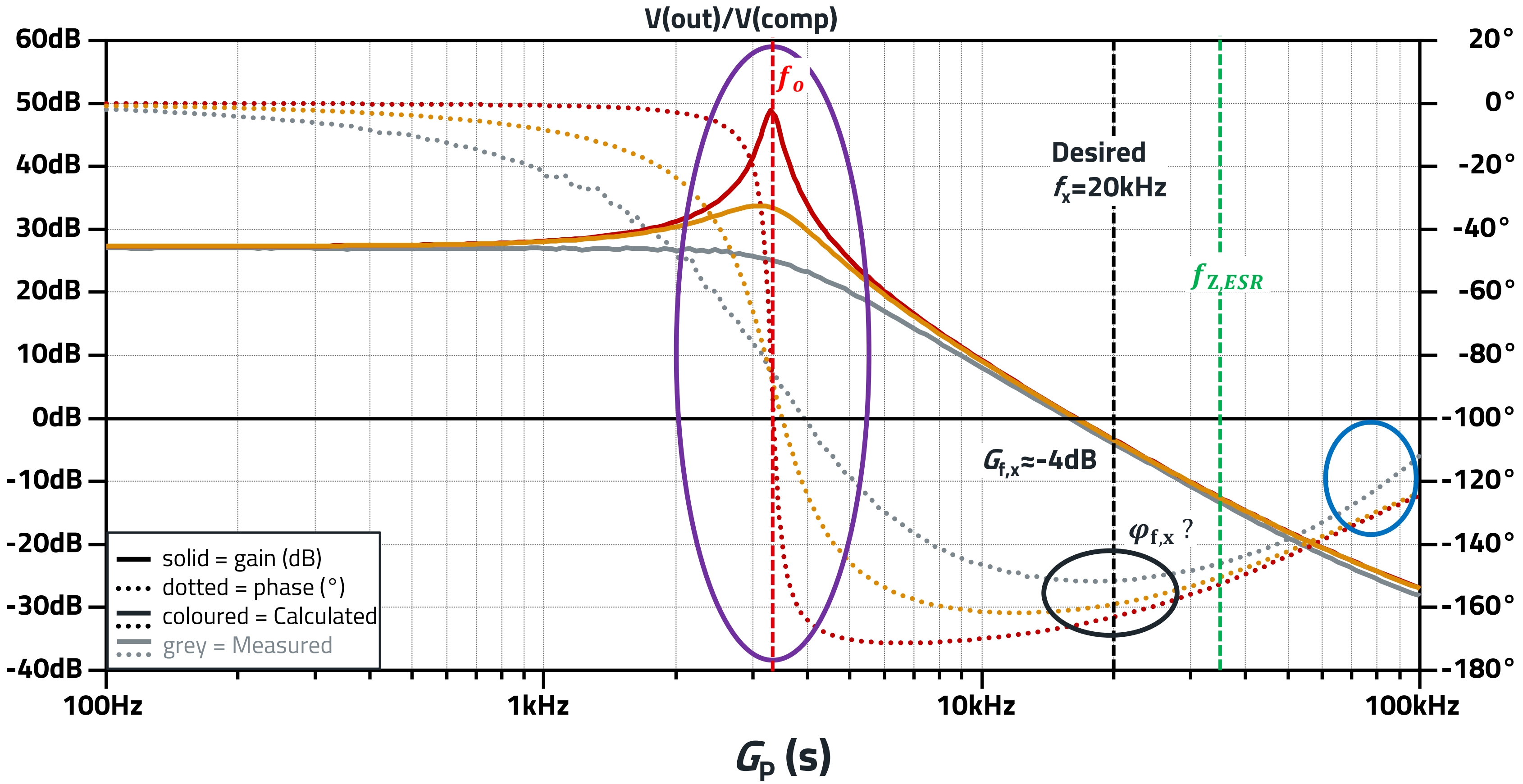
➢ $Q_0 = \frac{1}{\omega_0 \cdot \left(\frac{L}{R_{Load}} + C_{Out} \cdot \left(R_{ESR} + \left(1 + \frac{R_{ESR}}{R_{Load}}\right) \cdot R_{Ldc}\right)\right)} = \frac{1}{20{,}88 \cdot 10^3 \frac{rad}{s} \cdot \left(\frac{10\mu H}{2{,}5\Omega} + 227{,}5\mu F \cdot \left(20m\Omega + \left(1 + \frac{20m\Omega}{2{,}5\Omega}\right) \cdot 75m\Omega\right)\right)} \approx \mathbf{1{,}86}$

As in real life, quality makes the difference!

TRANSFER FUNCTION OF THE PLANT

Bode plot – calculation and measurement (model accuracy)

- The model accuracy depends heavily on the parasitics included
  - Red: Simplified transfer function (Laplace transform) from the slide before
  - Yellow: Transfer function (Laplace transform) including the damping effect and the natural frequency shift of the double pole due the ESR and DCR (inductor)
  - Grey: Measurement of the plant transfer function using RTA4000 Oscilloscope.
  - ➢ The quality is strongly influenced by the parasitic resistances simulated
  - ➢ If you start from the simplification, you get an overcompensated control loop
  - ➢ The ESL has an effect at higher frequencies

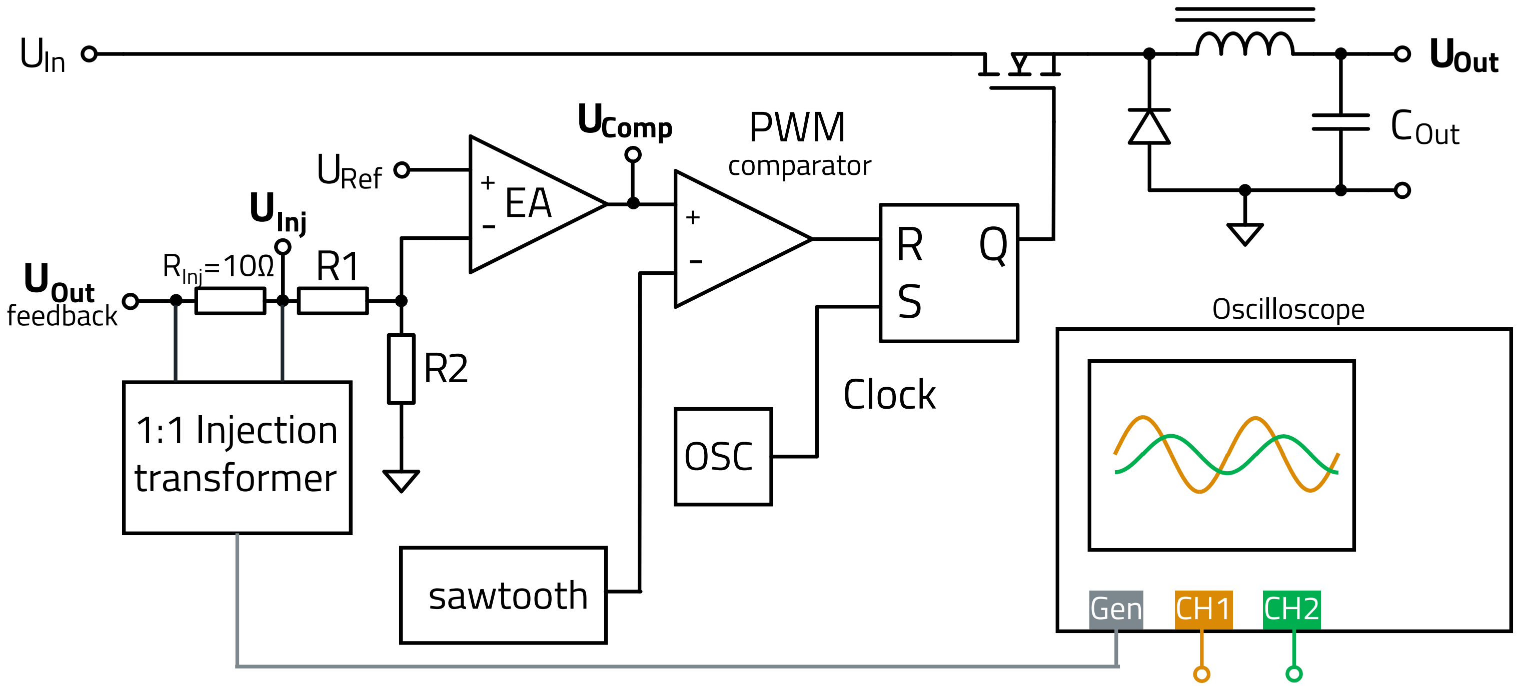


TRANSFER FUNCTION OF THE PLANT

Frequency response measurement - Bode plot

- A small injection resistor is inserted into the feedback voltage divider
  - ➢ The injection resistance is small compared to the series resistance of R1 (4,32kΩ) and R2 (825Ω)
  - ➢ The test signal (sinusoidal frequency sweep) is fed in via the injection resistor with an injection transformer

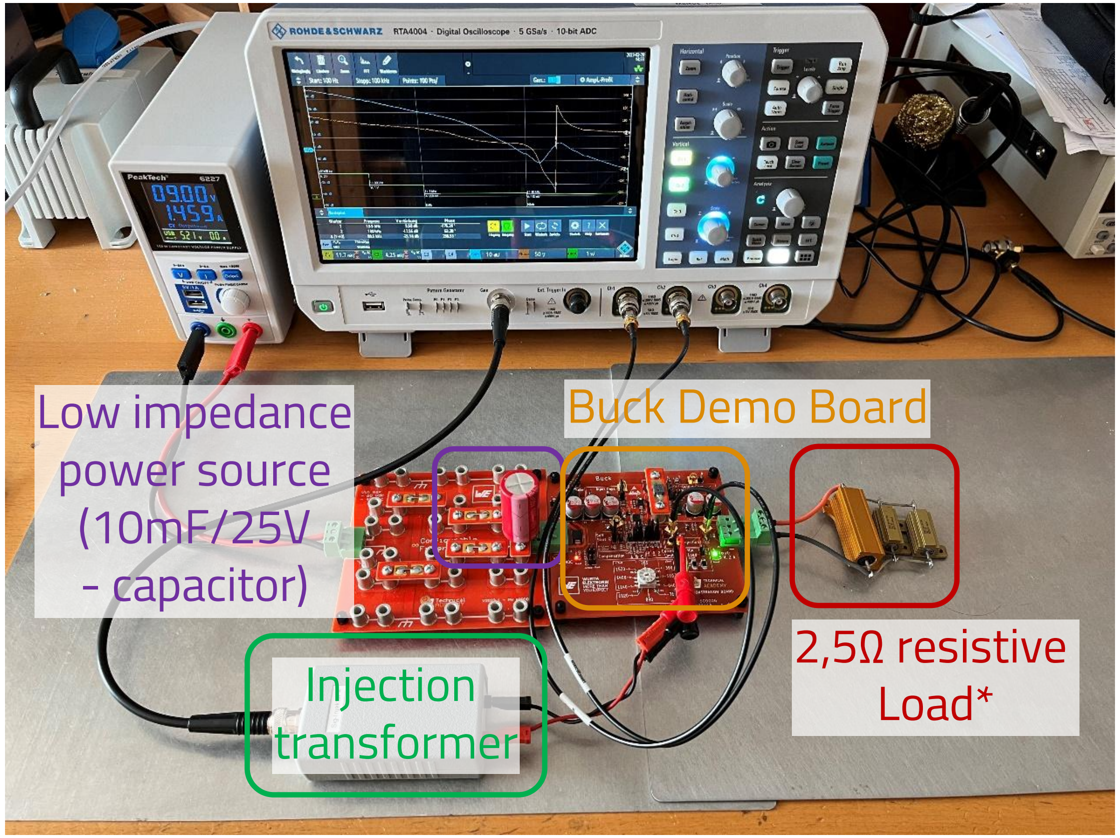
- The oscilloscope plots the gain and the phase by measuring the signal input/output ratio at each test frequency
  - ➢ Depending on which input/output ratio is evaluated, the plant, compensator or open loop transfer function can be determined



Transfer function		Input - CH1	Output - CH2
Plant	$G_P(s)$	U_{Comp}	U_{Out}
Compensator	$G_C(s)$	U_{Inj}	U_{Comp}
Open loop	$G_{OL}(s)$	U_{Inj}	U_{Out}

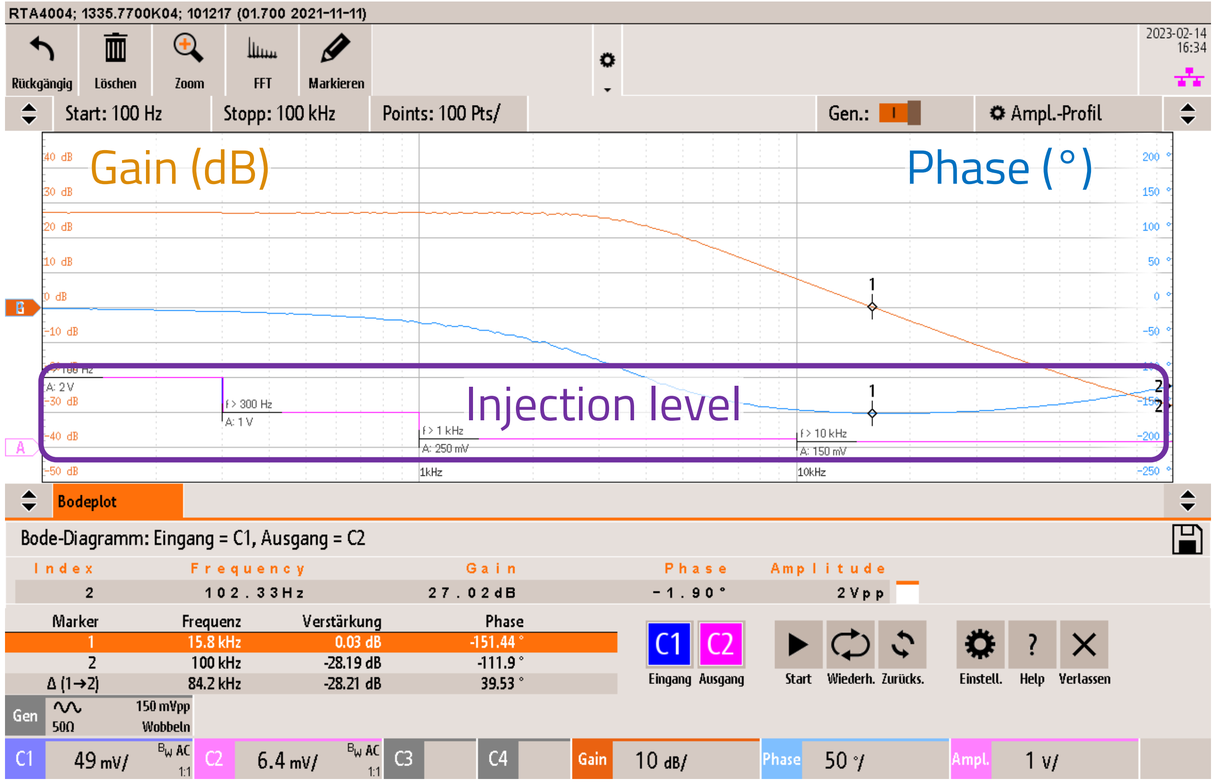
TRANSFER FUNCTION OF THE PLANT

Frequency response measurement – Bode plot with RTA4004-K36 (Rohde&Schwarz)



Test setup

*An resistive load is mandatory for frequency response analysis because a controlled electronic load affects the measurement



Measurement

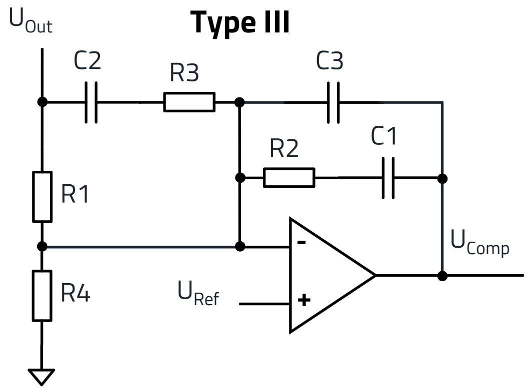
*The injection level must be chosen carefully:
Too big: wrong result (not the small signal behaviour)
Too small: bad SNR (noisy)

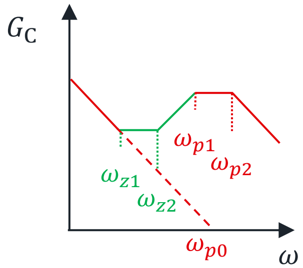
COMPENSATOR DESIGN

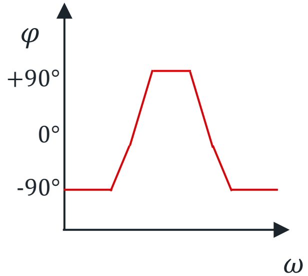
COMPENSATOR DESIGN

Type III - Compensator

- 1 pole of origin (integrator)
  - High gain at low frequencies
  - Small static error
- 2 poles & 2 zeros
- Gain and phase at crossover frequency can be affected
- Phase boost up to 180°
  - Due 2 zeros
  - Suitable for voltage mode
- Commonly used for voltage mode







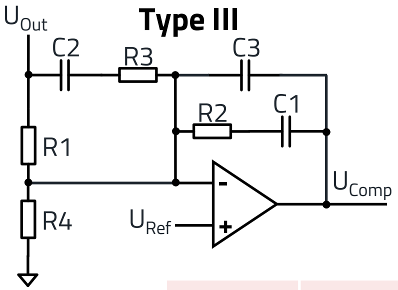
- $$G_C = (-)\,\frac{\omega_{P0}}{s}\cdot\frac{\left(1+\frac{s}{\omega_{Z1}}\right)\cdot\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\cdot\left(1+\frac{s}{\omega_{P2}}\right)}$$

- **Type III:**
  - $\omega_{Z1} = \frac{1}{R2 \cdot C1}$
  - $\omega_{Z2} = \frac{1}{C2 \cdot (R1 + R3)}$
  - $\omega_{P0} = \frac{1}{R1 \cdot (C1 + C3)}$
  - $\omega_{P1} = \frac{(C1 + C3)}{R2 \cdot C1 \cdot C3}$
  - $\omega_{P2} = \frac{1}{R3 \cdot C2}$

COMPENSATOR DESIGN

Type III - Compensator - Design example*

*Based on the plant transfer function of the Buck Demo Board



- Compensator A (R1=4,33kΩ; R2=10kΩ; R3=82Ω; C1=4,7nF; C2=4,7nF; C3=3.3nF)
- Compensator B (R1=4,33kΩ; R2=2.55kΩ; R3=82Ω; C1=10nF; C2=4,7nF; C3=1.5nF)*
- Compensator B + C (R1=4,33kΩ; R2=2.55kΩ; R3=82Ω; C1=10nF; C2=9,4nF; C3=1.5nF)

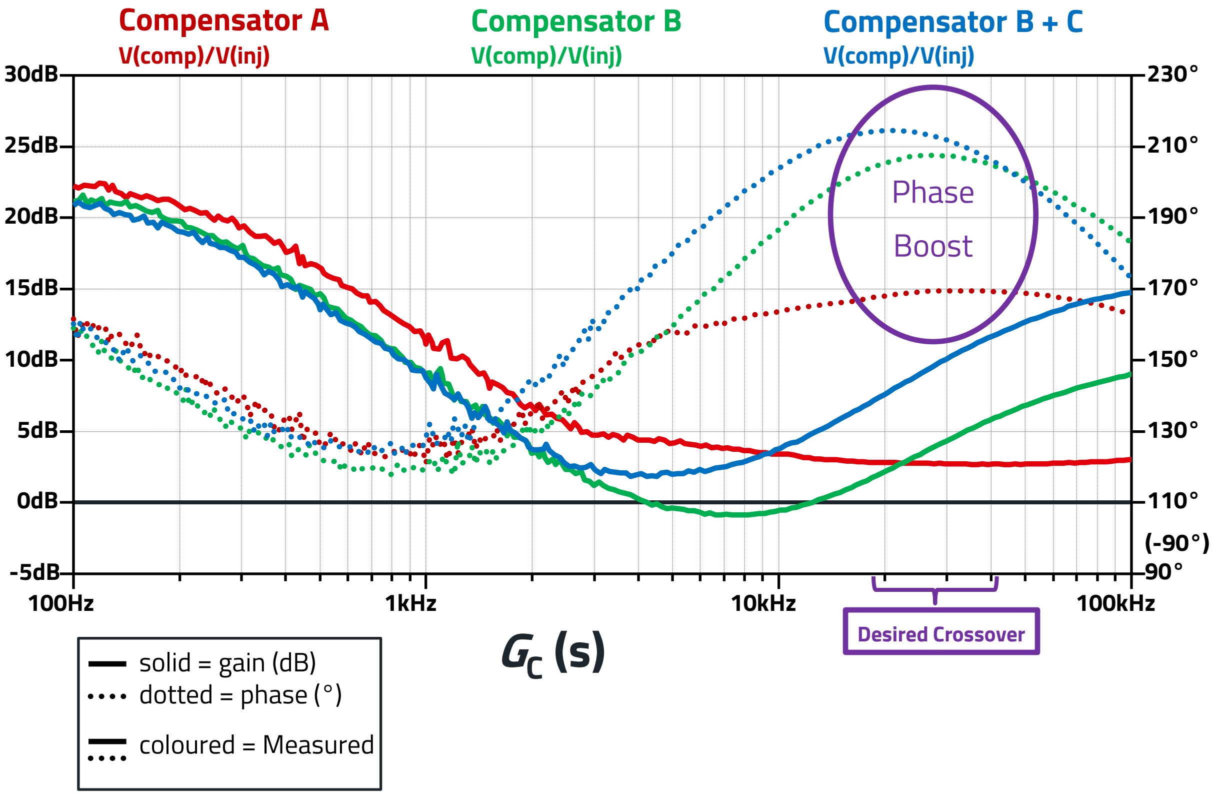
	Comp A	Comp B	Comp B+C
R1	4.33E+03	4.33E+03	4.33E+03
R2	1.00E+04	2.55E+03	2.55E+03
R3	8.20E+01	8.20E+01	8.20E+01
C1	4.70E-09	1.00E-08	1.00E-08
C2	4.70E-09	4.70E-09	9.40E-09
C3	3.30E-09	1.50E-09	1.50E-09

	Comp A	Comp B	Comp B+C
fz1	3387.99	6244.54	6244.54
fz2	7679.04	7679.04	3839.52
fp0	4596.87	3197.82	3197.82
fp1	8213.32	47874.78	47874.78
fp2	413169.87	413169.87	206584.94

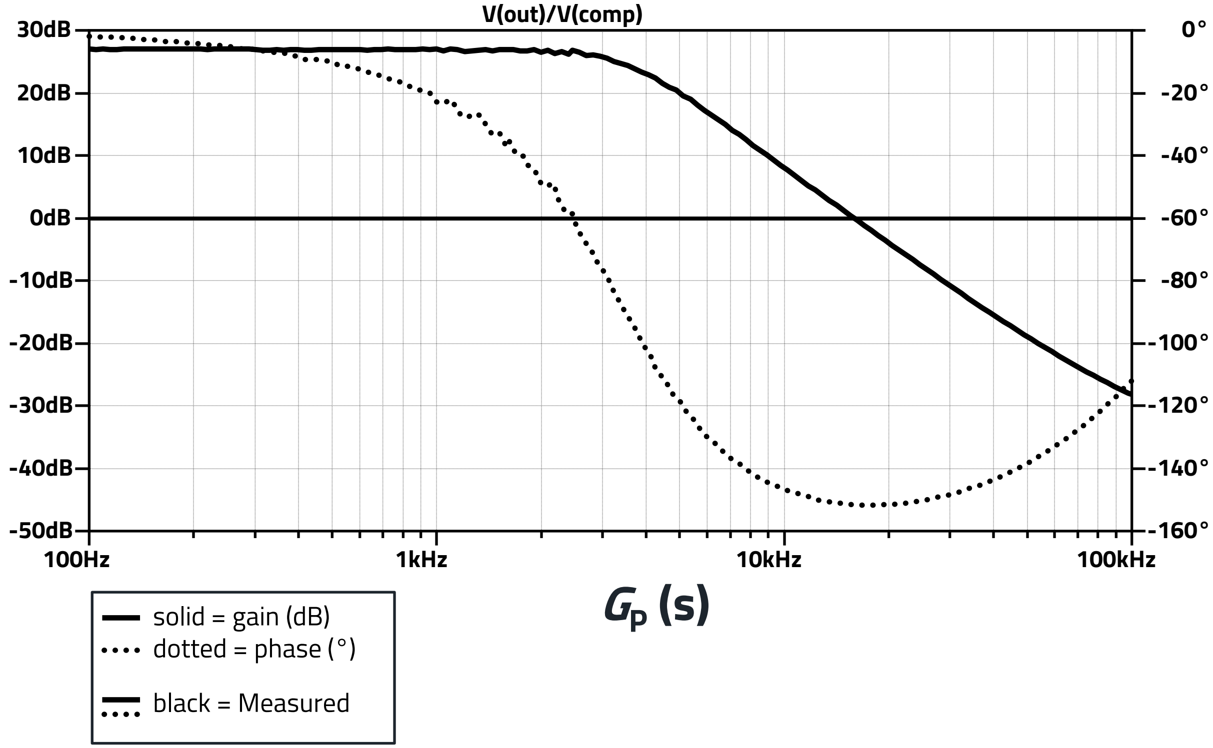
- Different locations for poles and zeros result in different open loop characteristics.
- Optimized performance going from compensator A -> B -> B+C.

STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Compensator and plant



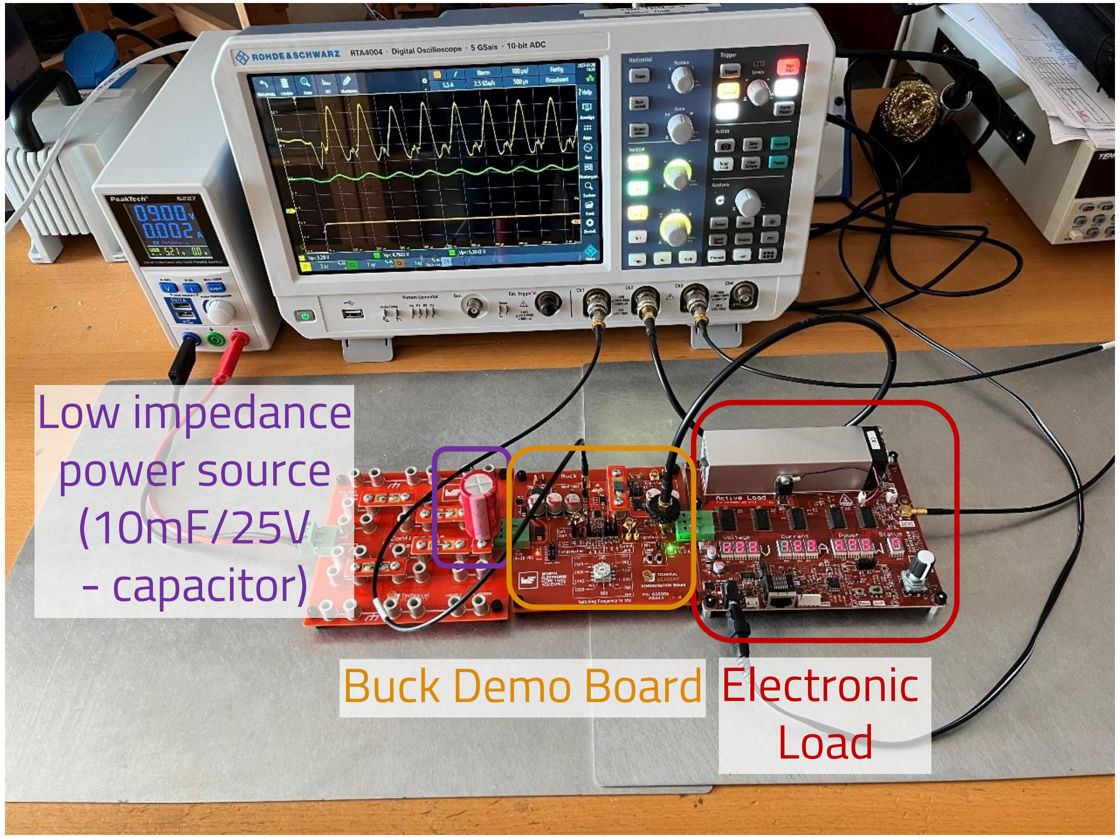
Bode plot – Compensator X



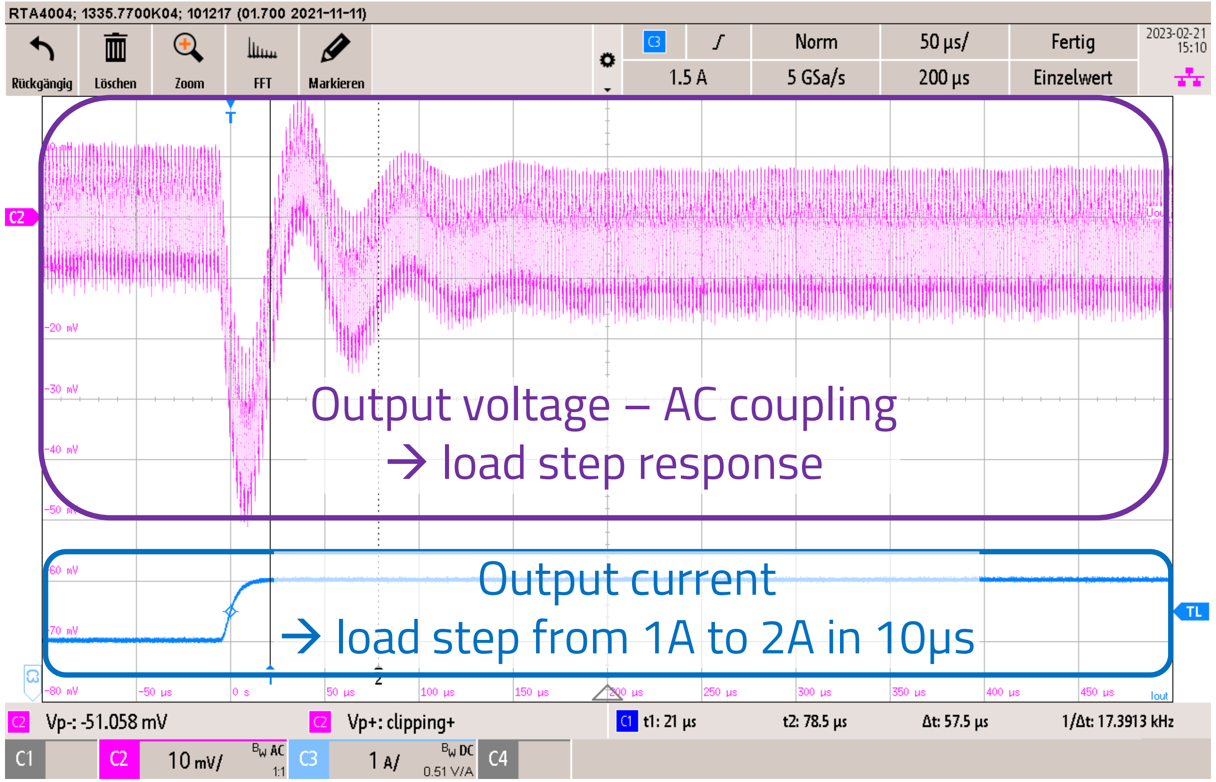
Bode plot – Plant: I_{Out}=2A / U_{In}=19V

STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Load step response - Test setup



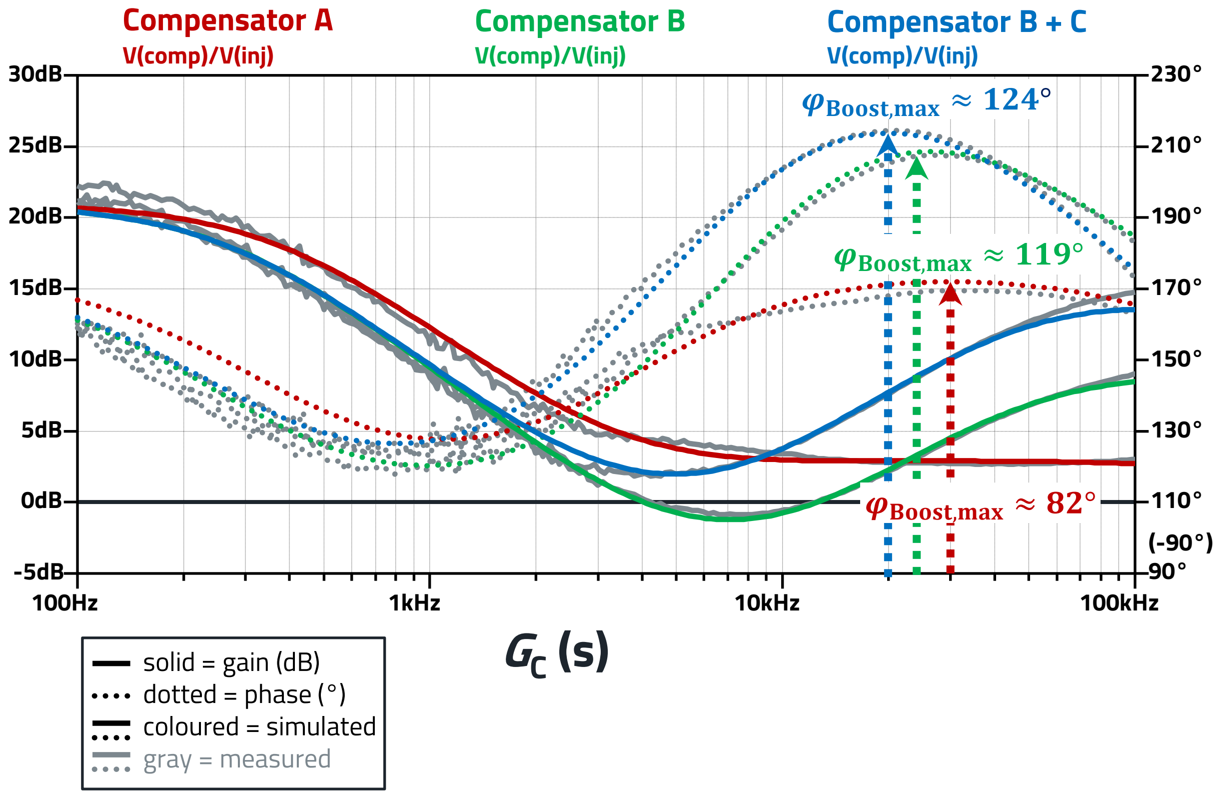
Test setup



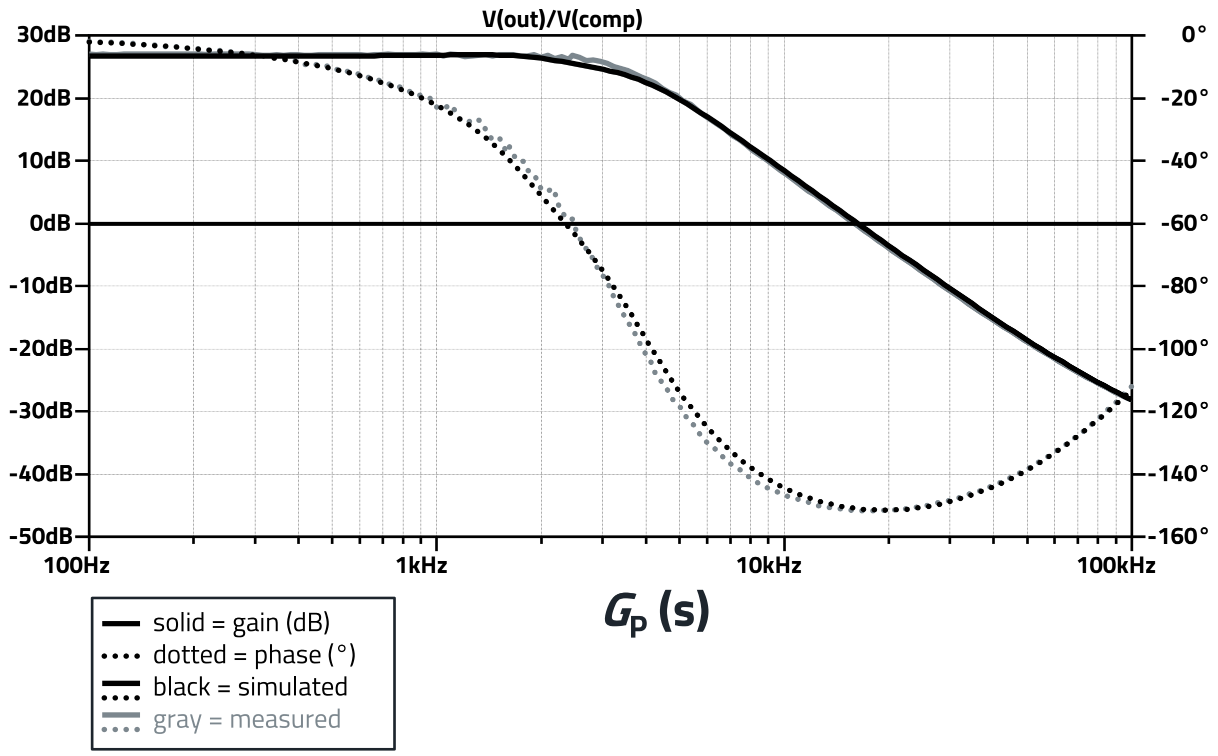
Measurement

STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Compensator and plant



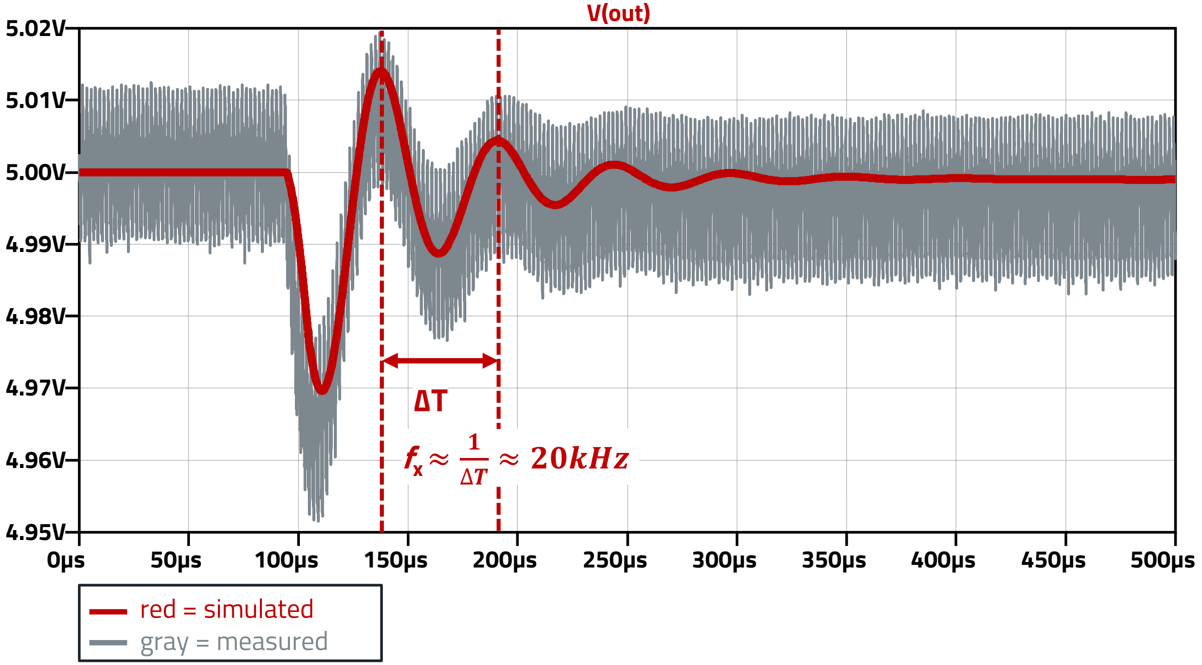
Bode plot – Compensator X



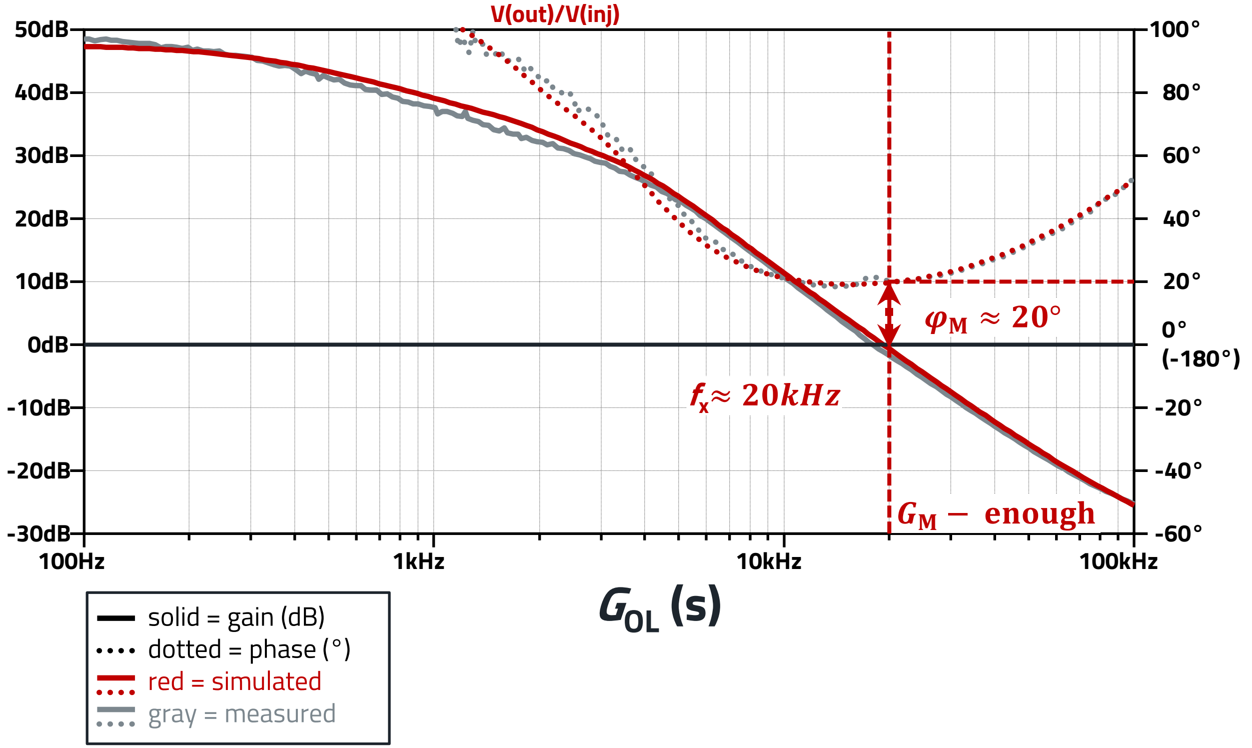
Bode plot – Plant: I_{Out}=2A / U_{In}=19V

STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Compensator A (R1=4,33kΩ; R2=10kΩ; R3=82Ω; C1=4,7nF; C2=4,7nF; C3=3.3nF)



Load step response: I_{Out} 1A → 2A / U_{In}=19V

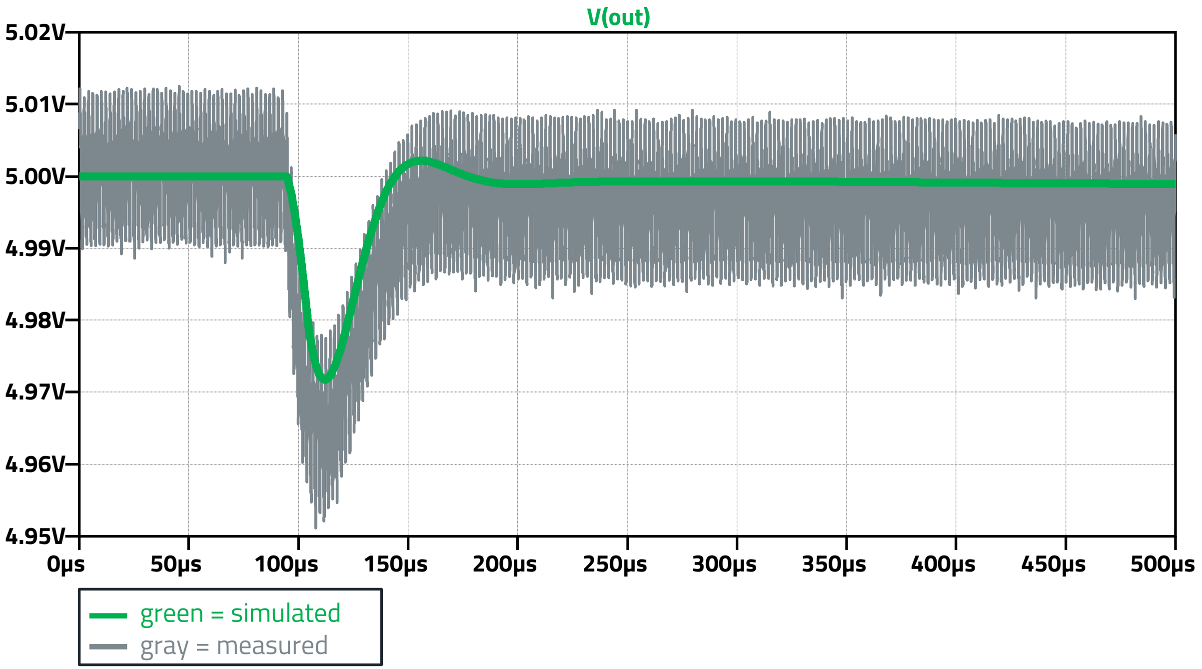


Bode plot - Open loop: I_{Out}=2A / U_{In}=19V

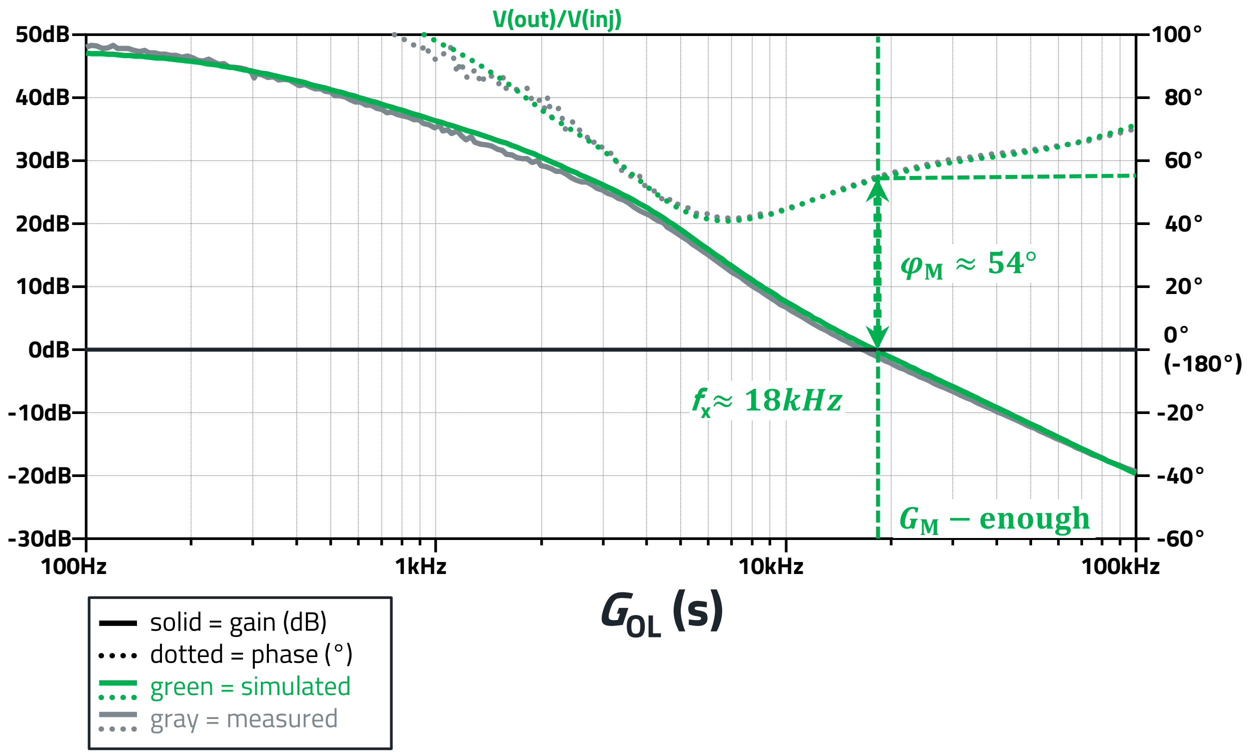
STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Compensator B (R1=4,33kΩ; R2=2.55kΩ; R3=82Ω; C1=10nF; C2=4,7nF; C3=1.5nF)*

*Design analyzed before



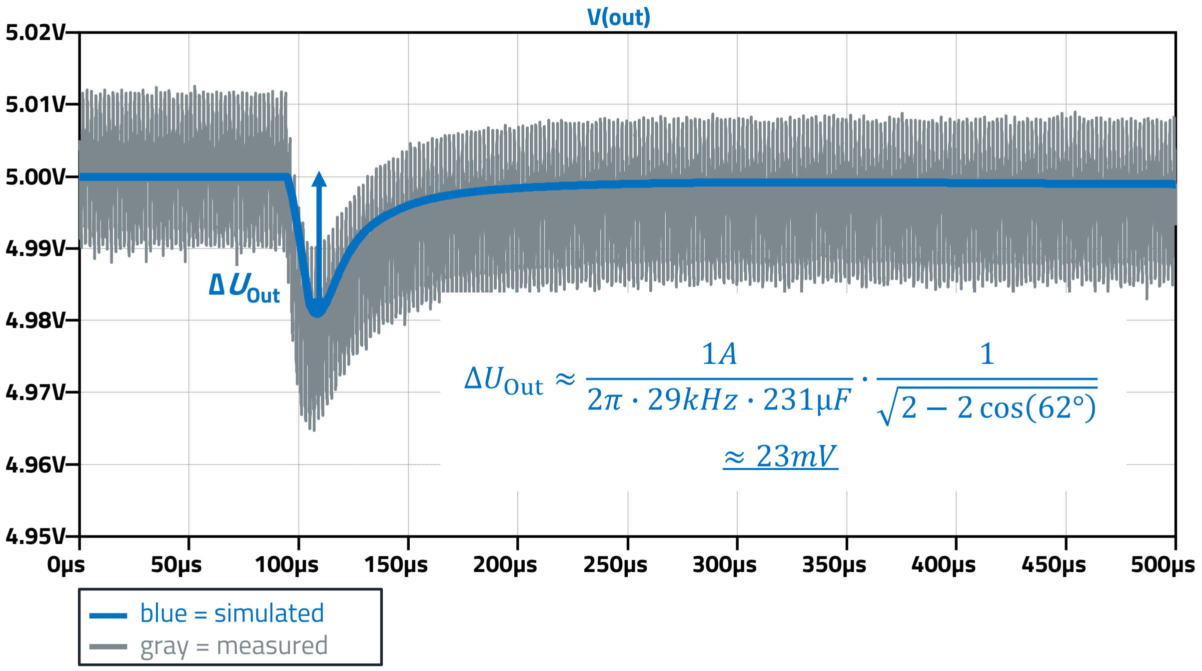
Load step response: I_{Out} 1A → 2A / U_{In}=19V



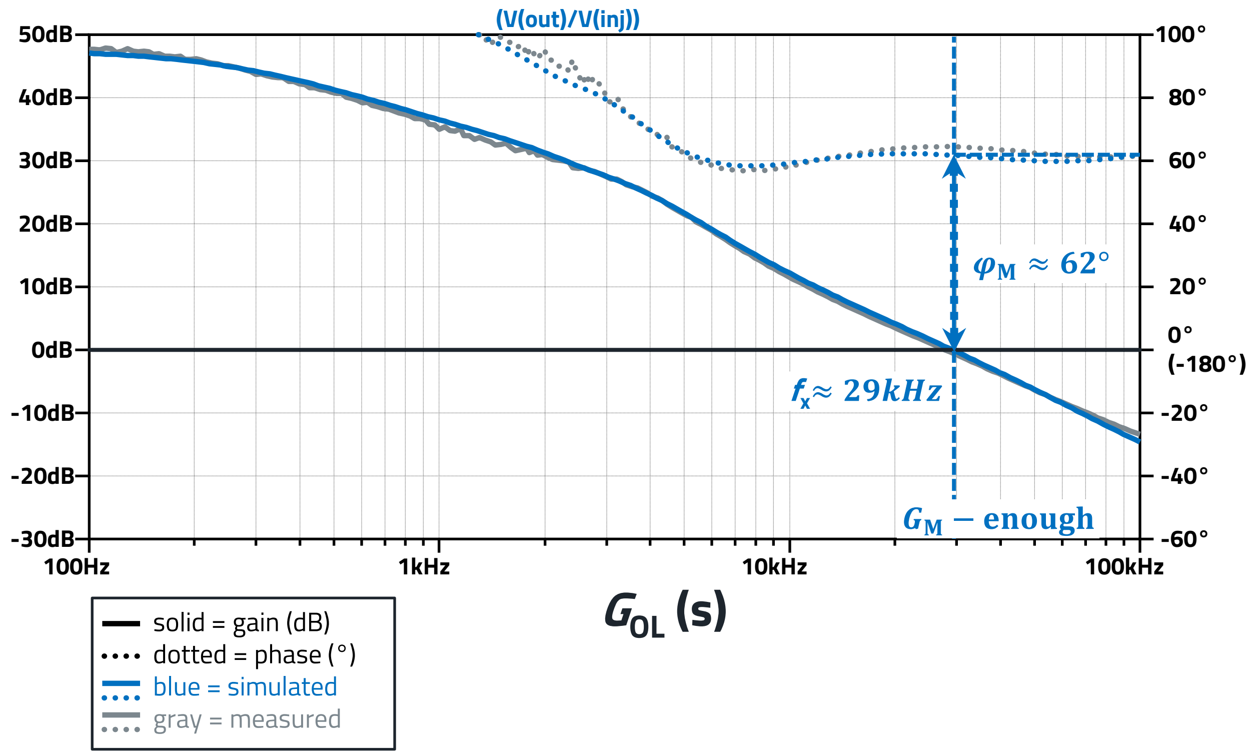
Bode plot - Open loop: I_{Out}=2A / U_{In}=19V

STABILITY THROUGH THE COMPENSATOR - BUCK DEMO BOARD

Compensator B + C (R1=4,33kΩ; R2=2.55kΩ; R3=82Ω; C1=10nF; C2=9,4nF; C3=1.5nF)



Load step response: I_{Out} 1A → 2A / U_{In}=19V



Bode plot - Open loop: I_{Out}=2A / U_{In}=19V