APPLICATION NOTE

ANP125 | Acoustic Effect of Harmonic Distortions caused by Aluminum Electrolytic Capacitors

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Abstract: This note reports a comparative study of total harmonic distortions (THD) caused by commercial electrolytic capacitors, as produced by Würth Elektronik eiSos as well as purpose-built items. The discussion about the audibility of distortions is made on the basis of human sound perception. This note arrives at the conclusion that capacitors do not add significant distortions to fundamental frequencies as they transfer signals. Modifications of the electrolyte or separation paper have almost no effect on the THD.

01. INTRODUCTION

There is an ongoing discussion within the audio engineering community about the sound quality of amplifiers concerning the audibility of signal distortions. [1], [2], [3], [4] Apparently, capacitors are suspected to be the source or at least a contributor to high-frequency distortions that influence the hearing impression. This work intends to add to the discourse about capacitors and their influence on distortions. The discussion about audibility of distortions involves not only the measurements of electrical characteristics but also their interpretation in terms of perception by the human hearing system.

This article reports a study of total harmonic distortions (THD) caused by commercial electrolytic capacitors, as produced by Würth Elektronik eiSos, as well as purpose-built items.

In order to find parameters that influence the THD, capacitors with different separation paper and electrolyte compositions have been investigated. Those sample capacitors were assembled at near mass production conditions at a production site and analyzed at the electrical laboratory of Würth Elektronik in Berlin.

In order to enable the reader to interpret the results, the article provides first an introduction into the field of human hearing and psychoacoustics before approaching the study of harmonic distortions in capacitors. It furthermore introduces results from model calculations to check the plausibility of the measured results.

To balance readability and technical thoroughness, the introduction into the technicalities of the THD and mathematical capacitor model development is not included in the main part but transferred to the Appendix.

02. HUMAN HEARING

This section provides an introduction into the human sound perception and how it can be quantified. It defines terms such as loudness, hearing threshold and summarizes the discussion about the prediction of the audibility of distortions.

The human ear can perceive sound waves in the frequency range between about 20 Hz (lower limit) and 16 kHz (upper limit). [5] A sound in this range (audible window) is therefore called audible sound. Sound below 20 Hz is called infrasound and sound above 16 kHz is referred to as ultrasound.

A graphical representation of the auditory sensation is achieved, if the sound pressure level is plotted over the frequency that is just audible, as in Figure 1. [6] The curves are called isophons and represent curves of equal loudness level, measured in units of phon. Isophons relate the sound pressure, measured in dB, to the volume levels. A sound with a volume level of 50 phon is perceived as just as loud as a 1 kHz tone with a sound pressure level of 50 dB.

Equal loudness level means that regardless of the frequency, each tone in the course of a curve is perceived as equally loud. The volume is, thus, a perception value (psychoacoustic), in contrast to the sound pressure, which represents a stimulus value. [7], [8], [9]

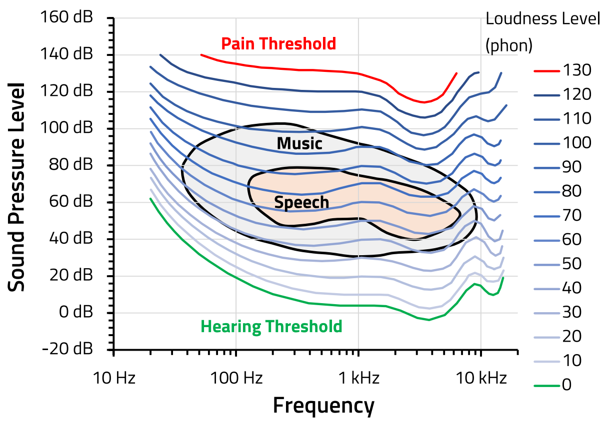

Figure 1: Auditory sensation area (DIN 45630)

In this plot, the lowest graph indicates the so-called threshold of hearing. This threshold applies to measurements with sine tones in a free sound field binaural hearing. The sound pressure level is related to the sound pressure 20 µP. With this definition, the sound pressure level at the hearing threshold at 1 kHz is 4 dB. The area for speech is much smaller than the auditory area. Even music encompasses only one sub-area of the entire hearing area.

The hearing threshold is obviously strongly frequency dependent. In the range between 2 kHz and 5 kHz the auditory sensitivity is greatest. In this range, the lowest sound pressure is sufficient for a hearing sensation. Below and above this range, hearing sensitivity decreases rapidly. The top curve represents the pain threshold. In this case, the sound pressure is large enough to inflict pain and, with longer exposure, causes permanent hearing damage.

At this point, it might become clear already that hearing is, by large degree, a matter of subjective perception. The determination of the volume, i.e. loudness, is complex and cumbersome.

So, the subjective measure of loudness is replaced by the objective measure of weighted sound pressure, as illustrated in Figure 2. Here the sound pressure is evaluated as a function of frequency with a filter characteristic, shown in its normalized form in Figure 2, which is approximately inverse to the isophons (curves of equal loudness) in Figure 1. [10] The normalized weighting filter curve in Figure 2 is based on the recommendation of the International Telecommunication Union (ITU). For the sake of clarity, Figure 2 also contains a curve indicating unweighted audible window.

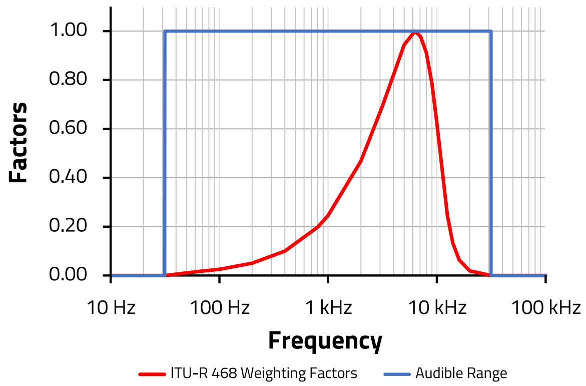

Figure 2: Graphic depiction of the filter factors for the unweighted audible range and weighing factors as suggested by ITU-R 468. [10]

Both, the weighted and unweighted filter curve, can be conveniently used to process audio signals.

The human dynamic range of hearing is large, spanning from 130 dB (threshold of pain) down to -9 dB (threshold of hearing). However, this wide span cannot perform these feats of perception at both extremes of the scale at the same time. [11] The ability to pick up a small distortion, superimposed to a fundamental or main signal very much depends on frequency range and on the complexity of the main signal. [2], [12], [13] Investigations indicate that for the complex signal of speech and music distortions of 2% to 5% can be introduced without being noticed by the listener. [11], [3] For single harmonic frequencies, it has been found that under laboratory conditions, human hearing is able to distinguish distortions by single harmonics, in the range of 0.3% down to 0,01% (at 4 kHz, range of highest sensitivity), relative to the fundamental frequency. [13]

To give an analogy for the corresponding differences in sound pressure level (see also appendix C): This would compare to the ability to distinguish the sound of a trumpet (sound pressure level of 120 dB) in a distance of about 10 km, while standing 1 m away from another trumpet with the same pressure level. [9], [7], [14] Although this analogy is a simplification and neglects frequency spectra of the trumpets as well as differences between fundamentals and the harmonic, anyone may now understand how sensitive human hearing can be and how small a distortion of 0.01% actually is.

The lowest THDs, for the first 10 harmonics, from above mentioned human hearing experiment range in the order of about 10% to 7%, depending on the fundamental frequency. (Appendix A provides more information about THD.) Hence, under defined conditions, the hearing is able to detect THDs as low as 7%, which corresponds to a change in sound pressure level of 20 dB.

There is evidence that the THD indeed is not always the most accurate measure (metric) to predict the audibility of distortions in complex waveforms. [1], [2], [3], [12] Multitoned signals or systems with disturbances at higher frequencies can be perceived very differently in terms of subjective disturbance. For instance, it was found that a sine-wave driven loudspeaker ("bad loudspeaker") with a THD of 2%, gives rise to a "Rub & Buzz" distortion, if the spectrum contains non-decreasing amplitudes of 0.3% above the 10th harmonic, i.e. at higher frequencies. [4] However, a loudspeaker ("good loudspeaker") with a higher THD of 6%, but decreasing higher harmonics amplitudes, exhibits a low "Rub & Buzz" distortion and, thus, a better sound. For those systems, it is recommended to use methods such as the GeddLee metric or the use of an n-th harmonic weighting factor $n^2/4$ to measure the psychoacoustic influence of the high-frequency distortion. [3]

Hence, the THD is a measure for systems that show low nonlinear disturbances that produce first harmonics below or about 1% and decrease to zero at higher harmonics. However, for larger nonlinear disturbances, with non-vanishing higher harmonics, the THD might not give a correct measure for the audibility of disturbances.

03. THD OF A CAPACITOR MODEL

In this section, a model is introduced in order to be able to check the plausibility of measured THD, presented in section 05. In appendix A, the concept of THD, which is merely technical, is reviewed and explained with examples.

Figure 3 shows a circuit, suitable to model the voltage and current behavior of a capacitor. It consists of an equivalent series resistance (ESR) R, a pure capacitance C and an equivalent series inductance (ESL) L, modeling the capacitor, as well as a voltage source and an ampere meter.

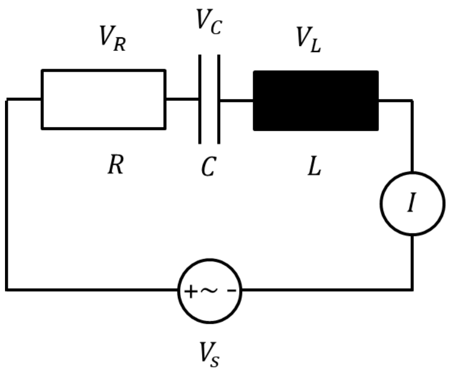

Figure 3: Capacitor model consisting of equivalent series resistance R, capacitance C, equivalent series inductance L, ampere meter and voltage source.

Appling Kirchhoff's rule to the circuit in Figure 3 leads to

$$V_s = V_R + V_L + V_C \qquad (1)$$

with V_s as the time-dependent voltage signal applied by the source, V_R as the voltage drop over the ESR, V_L as the voltage drop over the ESL and V_C as the voltage drop of the pure capacitance. Explicit mathematical expressions of the solution of equation (1) are of a purely technical nature and are given in appendix B. Since the capacitor is not causing any harmonics the THD is zero. (see appendix A for details)

This changes, if the constant capacitance is replaced by a voltage-dependent term, i.e. non-linear capacitance. Based on measurements the voltage-dependent capacitance change is estimated at about 1% as illustrated in Figure 4. With the voltage-dependent capacitance, the model produces higher harmonic oscillations as it is subjected to an oscillating voltage signal.

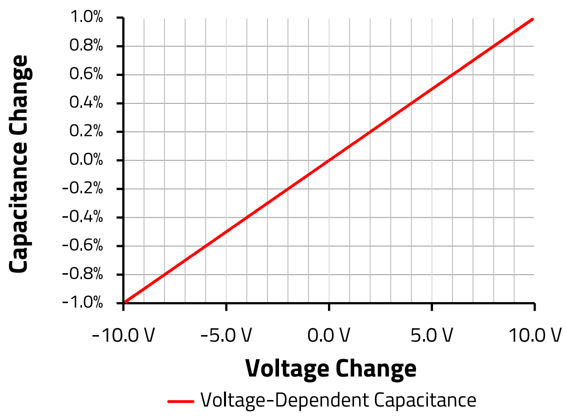

Figure 4: Dependence of the relative capacitance on the applied voltage change. Represents an upper estimate of actual measured capacitance-voltage dependence.

The result of the Fourier analysis, given in Figure 5 provides, the normalized amplitudes of the harmonics for a specific fundamental frequency $\omega_1 = 2\pi f_1$, as given in equation (5). The calculation is made with C = 470 µF, R = 0.1 Ω and $L = 50\ 10^{-9}$ H. The amplitude of the first harmonic is about 0.086%. Higher harmonics rapidly decline in amplitude below 0.001%. In accordance to the model, the frequency response yields a THD of 0.086%.

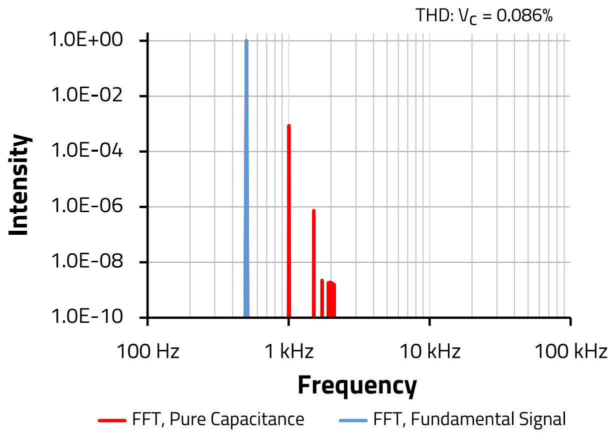

Figure 5: Calculated frequency spectrum of capacitor model with voltage dependent capacitance, as shown in Figure 4. The fundamental frequency is f_0 = 500 Hz

The model might not predict exactly the amplitudes of harmonic frequencies, since such amplitudes depend on the voltage dependence of the capacitor, which in this case is only estimated. It should, however, provide an approximation of the actual amplitude of the THD and it shows that the voltage-dependent capacitance leads to non-zero harmonic distortion. The first harmonics decline rapidly and have THD well below 0.1%, which would suggest that this distortion is at the threshold of audibility or below, while the latter is more likely.

In the last two sections, the subject of distortions and their audibility has been revised and studied on the basis of literature and model calculation. In the further course, this article presents and discusses measured harmonic distortions as well as electrical characteristics of different electrolytic capacitors.

04. EXPERIMENTAL DETAILS

The study comprises capacitors, which are commercially available at eiSos Würth Elektronik (see Table 1) as well as purpose-built capacitors produced at the production site (see Table 2). The samples in Table 2 have been prepared at the production site under near-mass production conditions to vary separation paper composition, separation paper density, separation paper thickness and electrolyte composition. The paper was supplied by Nippon Kodoshi Corporation and the main components of the electrolyte are γ-butyrolacton, dimethyl carbonate and ammonium tetraborate tetrahydrate. (To protect the intellectual properties of Würth Electronic (WE) further preparation details may not disclosed without the signing of a non-disclosure agreement)

Sample Name	Series	Capacitance [µF]	Rated Voltage [V]	Part Number
WE, ASLI	WCAP-ASLI	470	10	865080253012
WE, AT1H	WCAP-AT1H	470	10	860240275007

Table 1: Overview of commercially available samples.

For samples P1, P2 and P3 only the paper has been changed to study its influence on the electrical characteristics. With the samples P4, P5 and P6 the influence of the water content in combination with different separation papers should be examined. All capacitors have the same dimensions and are surface-mount devices.

Sample Name	Paper			Electrolyte	
	Thickness [µm]	Density [g/cm³]	Main Fiber Component	Water Content [%]	Conductivity [mS/cm]
P1	40	0.45	Manila Hemp	18 - 21	14
P2	40	0.4	Manila Hemp	18 - 21	14
P3	40	0.35	Bast	18 - 21	14
P4	40	0.4	Manila Hemp	49 - 55	32
P5	30	0.66	Synthetic	18 - 21	14
P6	40	0.35	Bast	49 - 55	32

Table 2: Overview of sample and variation of materials.

The measurements were conducted with the impedance analyzer Alpha-AK High Performance Frequency Analyzer connected to the frontend POT/GAL, Electrochemical Test Station (15 V, 10 A) from Novocontrol in a four-terminal Kelvin configuration. The analyzer control as well as the harmonics acquisition were carried out with WinDETA Software (V 6.02) from Novocontrol.

Any subsequent data evaluation as well as the model calculations have been performed with Wolfram Mathematica 11. The ESR and capacitance values have been obtained by fitting a Resistance-Capacitance-Inductance model to the impedance data.

Before the measurements, each sample has been pre-poled for 20 minutes at its rated voltage (10 V) with HMP4040 programmable power supply from Rohde & Schwarz. ESR and Capacitance were measured with a dc offset of 1 V and a probing ac signal amplitude of 0.1 V. For the THD measurements, the DC offset was 6 V and the probing ac signal amplitude was 1.5 V. For each capacitor type, at least five capacitors have been measured.

05. MEASURED THD OF 470 µF ELECTROLYTIC CAPACITOR

This section explains the frequency analysis for the example of an electrolytic capacitor from Würth Electronic with a capacitance of 470 µF (865080253012). It covers the calculation of the THD and introduces the mean-THD as a figure of merit for the signal distortion at the capacitor.

For the measurement of the harmonic ac voltage components a probing voltage signal, with a specific frequency f_1, is applied to the capacitor, which in turn causes a current that leads to charging/discharging of the capacitor. The voltage at the capacitor, measured with separate probing leads, can now be analyzed with a Fast Fourier Transformation (FFT) to obtain a frequency spectrum of the measured voltage response of the capacitor. The applied frequency f_1 is the lowest possible frequency and thus the fundamental frequency, in this measurement.

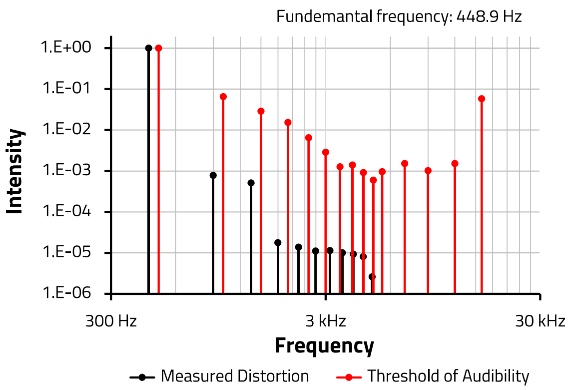

Figure 6: Measured frequency spectrum of 470 µF aluminum electrolytic capacitor (WCAP-ASLI, 865080253012) at fundamental frequency of voltage signal is 448.9 Hz. Also shown, threshold of audible distortions as determined in psychoacoustic experiment for a fundamental at 500 Hz. [13]

The measured frequency spectrum of a capacitor with the fundamental frequency of 448.9 Hz, in Figure 6, shows a steep decline in amplitudes for higher harmonics, which is typical for all studied capacitors and excitation frequencies. The amplitudes of the first two harmonics are decreasing to values well below 0.1% compared to the fundamental signal. The higher-order harmonics attain values in the order of 0.001% and below. All harmonic amplitudes are well below the threshold of audibility, which is also plotted in Figure 6. The displayed hearing threshold values have been determined in a separately conducted psychoacoustic experiment at a fundamental at 500 Hz. (see also appendix C for low-frequency example) [13]

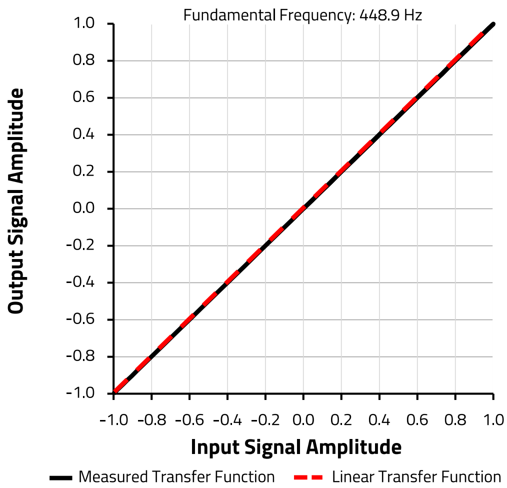

Figure 7: Linear and measured transfer function of a 470 µF aluminum electrolytic capacitor (WCAP-ASLI, 865080253012). Fundamental frequency of voltage signal is 448.9 Hz.

As a result of the low amplitudes of the harmonics the corresponding transfer function, given in Figure 7, shows a nearly perfect linear behavior. The corresponding amplitude of the superimposed disturbance is about a factor of 1000 smaller than the fundamental itself. Hence, it can be argued that the capacitor is in good approximation a linear system.

Other measurement quantities like the GeddLee metric or metrics that involve the emphasis on higher harmonics will not provide further information. [3] The GeddLee metric involves the second derivative of the transfer function, which for capacitors is in good approximation zero and higher harmonics beyond the 10^{th} harmonic show declining amplitudes below 0.001%, which can be neglected.

As discussed in section 02, under these conditions the THD is a suitable measure for the comparative study of distortion.

The order of magnitude of the first harmonic as well as the steep decline for higher harmonics agrees with the model calculation shown in Figure 5. The THD (details given in appendix A) calculated on the basis of the Fourier transformation in Figure 6 yields a value of 0.078%, which compares roughly to the theoretical value of 0.086%, calculated in section 03. Taking the model calculations, given in section 03 as a reference, the frequency response of the capacitor is as expected.

The THDs for fundamental frequencies in the range of 1 Hz to 1 MHz, given in Figure 8, show variations in the range of 0.001% to 0.4%. This THD spectrum shows by trend lower THD values at low frequencies and larger THD values at high frequencies. Within the audible range, displayed in Figure 2, the values are around 0.05% or so.

At this point, we want to refrain from further discussion of the spectra but utilize mean values to assess the quality of a capacitor. To obtain a figure of merit for the frequency distortion within the audible range $\Delta f = f_2 - f_1$ we numerically calculate the average

$$THD_{Mean} = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} THD(f)\, df \qquad (2)$$

as well as the weighted average

$$THD_{ITU} = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} W_{ITU}(f) \times THD(f)\, df \qquad (3)$$

on the basis of the human hearing sensitivity $W_{ITU}(f)$. For the given example, the averages are: THD_{Mean} = 0.097% and THD_{ITU} = 0.017%. The weighted average is lower, since human hearing is less sensitive below 1 kHz and above 10 kHz.

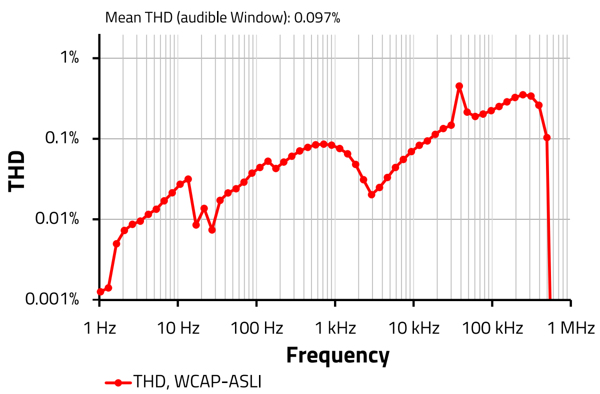

Figure 8: THD of 470 µF aluminum electrolytic capacitor (WCAP-ASLI, 865080253012), measured at different fundamental frequencies in a range from 1 Hz to 1 MHz.

As discussed in section 02 the hearing threshold for distortions is about 7%, especially if the amplitudes of the harmonics decline towards zero as they do in the shown example. Hence, the harmonic distortions of the studied capacitor are by orders of magnitudes below the auditory perception.

Based on these results it is unlikely that capacitors are a significant source of distortions, if used for signal coupling (series connection) or decoupling (parallel connection) in audio applications. The transfer functions of amplifiers show by design higher nonlinearities than capacitors. [15], [16], [17] It cannot be entirely ruled out that clipping or other strong non-linear effects of power amplifiers may lead to accentuation of capacitor-related minuscule distortions up to the point where they become clearly audible.

The further investigation of such effects exceeds the scope of this work. Future studies of this effect would have to separate the high distortions incited by the clipping itself from those caused by the capacitor.

06. COMPARATIVE ANALYSIS OF CAPACITORS

Section 05 exemplified the Fourier analysis on a single capacitor and introduces the average THD values THD_{Mean} as well as THD_{ITU} as a measure for the frequency distortion within the audible range. In the following section, the THD_{Mean} as well as THD_{ITU} are used to quantify the influence of different electrolyte compositions and separator paper on the signal distortion.

Figure 9 and Figure 10 show the capacitance and ESR values of the different capacitors, respectively. As stated in section 04, the samples from Würth Elektronik (WE) are commercially available. For the samples P1 to P6 the water content of the electrolyte as well as the paper density has been systematically varied. All samples have rated capacitances of 470 µF, within the customary tolerances of ±20%, as illustrated in Figure 9.

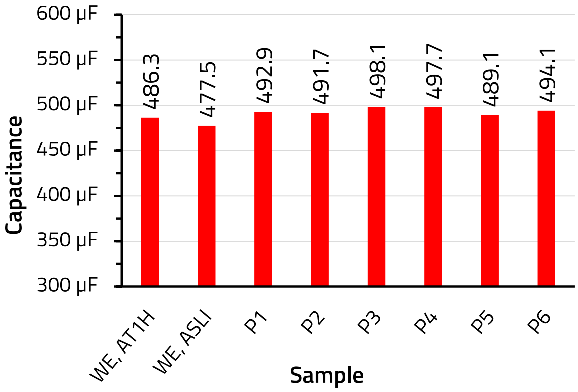

Figure 9: Average values of measured capacitances.

As displayed in Figure 10, the ASLI samples have the lowest ESR value among the commercially available products, since it is from a designated low-loss product line. The samples P1 to P3 show a further decrease of the ESR compared to the ASLI sample as the density of the separator paper is decreased. This effect is related to the increasing mobility of the electrolyte, due to the increasing pore size of the separator paper.

For sample P4 the paper and the electrolyte were chosen with the aim to obtain a further decrease of ESR. For sample P6 the paper density has been further lowered, resulting in a further ESR decrease in comparison to sample P4. P5 illustrates the effect of a high-density paper on the ESR, which has a larger ESR compared to samples P1 to P3.

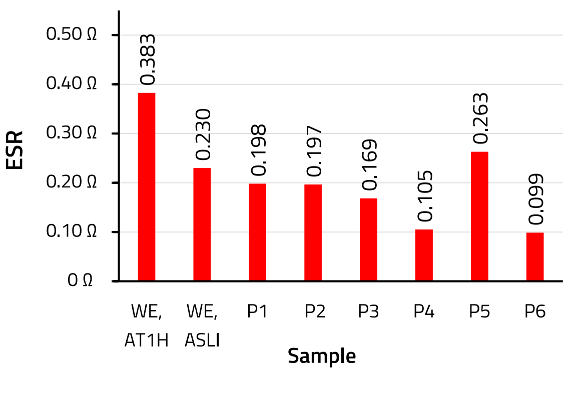

Figure 10: Average values of ESR.

Indeed, there is more to say about the influence of electrolyte and separator paper on the ESR, however, at this point, it is actually sufficient to acknowledge the presented capacitors constitute a broad variety of electrolyte and separator paper. Interestingly, the effect of these material variations and differences in ESR values on the THD_{Mean} as well as THD_{ITU}, plotted in Figure 11, is practically neglectable, as the mean range, indicated by the error bars, suggests. Generally, it can be said that THD_{ITU}, which takes the frequency-dependent human hearing sensitivity into account, is one order of magnitude lower than THD_{Mean}.

The THD_{Mean} and THD_{ITU} of the AT1H compared to P1 is nearly unchanged, although the ESR value of P1 is markedly decreased compared to AT1H.

The effect of material variations on the THD can be seen in the comparison of samples P3 and P4. Here paper and electrolyte have been changed with no noticeable effect on THD_{Mean} and THD_{ITU}. Similar could be found in the comparison of samples P2 and P3, which contain different separator papers but the same electrolyte.

A different THD_{Mean} can be found for samples P5 and P6. The sample P5 with the high ESR and the large paper density has a lower THD_{Mean} than P6 with a low ESR and low paper density. This could suggest an inversely proportional relation between ESR and THD_{Mean}. However, that inverse proportionality cannot be confirmed for samples AT1H and P5. The THD_{Mean} for those two is nearly the same. However, when focusing at the frequencies of high audible sensitivity, the changed THD_{ITU} for the above-discussed AT1H and P5 would suggest a direct proportionality.

Hence, it is difficult to find the cause for the small variations of THD_{Mean} and THD_{ITU}. In fact, as the error bars indicate, the differences between most of the measurements are of low statistical significance. Thus, the variations of separation paper and electrolyte have not influenced the THD in a significant way. This result is corroborated by another study, made on a wide range of capacitors, which also found no significant differences in the degree of distortions. [17]

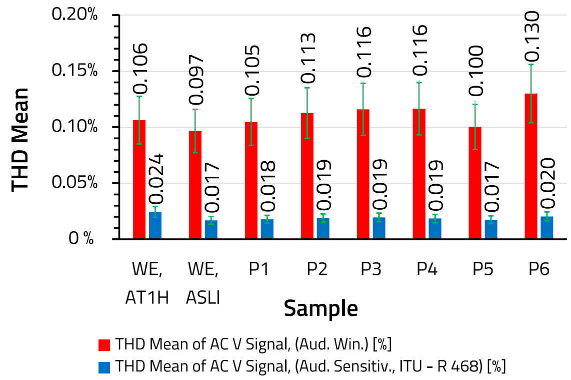

Figure 11: Measured THD_{Mean} (audible window) and THD_{ITU} (audible sensitivity) values. Error bars indicate the mean range of values.

07. CONCLUSION

This study has investigated the ESR, capacitance and THD of nine different capacitor types. While the commercial capacitors as well as the material variations of capacitors P1 to P6 have shown changes in ESR, none of them has shown significant changes of the THD_{Mean} as well as THD_{ITU}. The plausibility of the THD value for a fundamental frequency of 500 Hz has been checked against model calculations.

The unweighted THD_{Mean} was about 0.1% and the weighted THD_{ITU}, which suppresses less audible harmonics in accordance with ITU-R 468, was about 0.02%. Even the THD_{Mean}, which includes THDs from less audible frequencies, suggests that the harmonics distortions are well below the threshold of audibility, which is at about 7%. In the range of the highest audible sensitivity, the THD values of THD_{ITU} are even 10 times smaller than the values of THD_{Mean}.

It can be concluded that the electrolytic capacitors do not add significant distortions to fundamental frequencies as they transfer signals and thus, in good approximation, can be considered as linear devices. Modifications of the electrolyte or separation paper have a neglectable effect on the THD. It is likely that other voltage-independent capacitor types and passive components in general will produce similarly low distortion amplitudes compared to the threshold of audibility.

Consequently, the choice of the non-linear devices, such as op-amps and diodes will have more effect on the distortion-related audio quality of the amplifier, i.e. the overall distortion characteristics, than the choice of the electrolytic capacitor.

One may speculate that maybe other effects such as soundwave-induced vibrations, transduced by the circuit board onto the capacitor assembly, could interfere with the audio signal and thus have an effect on the audio quality. This, however, cannot be answered by this study but could be subject to further research.

Acknowledgment

This research would not have been possible without the R&D team at the production site. Special thanks to the technical expert Eric Fischer as well as Jon Izkue-Rodriguez at the Würth Electronik Competence Center Berlin, who provided technical support.

A. Appendix, Review of THD and Fourier Transformation

From an electrical point of view, an audio signal is an oscillating current/voltage often in the human audible range. Audio signals may be characterized by parameters such as frequency bandwidth and signal level. With analog audio signals, the signal level, usually given in decibels (dB), corresponds directly to the amplitude of the electrical voltage, which in turn is proportional to the sound pressure. Any sound signal can be represented as a superposition of harmonic oscillations. Each of the oscillation has its specific frequency and amplitude.

The mathematical process of discretizing arbitrary signals into harmonic oscillations is performed with the Fourier transform (or Fourier analysis). Any time dependent signal F(t) may be represented by a Fourier series

$$F(t) = a_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \cos(k\omega_0 t) \qquad (4)$$

Where a_0 is a constant offset, ω_0 the fundamental frequency, k is the index number, and a_k as well as b_k are Fourier coefficients. The first harmonic oscillation (k=1) is also referred to as the fundamental or fundamental signal. It is the signal with the lowest harmonic frequency in the sum of harmonically related signals.

Fourier coefficient may be used to calculate the total harmonic distortion (THD) of a signal

$$THD = \sqrt{\frac{\sum_{k=2}^{\infty} (a_k^2 + b_k^2)}{a_1^2 + b_1^2}} = \sqrt{\frac{\sum_{k=2}^{\infty} |c_k|^2}{|c_1|^2}} \qquad (5)$$

With c_k as the amplitude of the k-th harmonic. The THD provides a measure of signal distortion in comparison to fundamental frequency signal.

To give an example, Figure 12 and Table 3 shows some commonly known waveforms and the calculated THD, respectively. Due to their mathematical description, it is not only possible to calculate the numerical THD values but also the analytical expressions of THD. While it might not always be possible to arrive at a relatively simple analytical expression, it is always possible to perform a Fourier analysis and to compute the THD value.

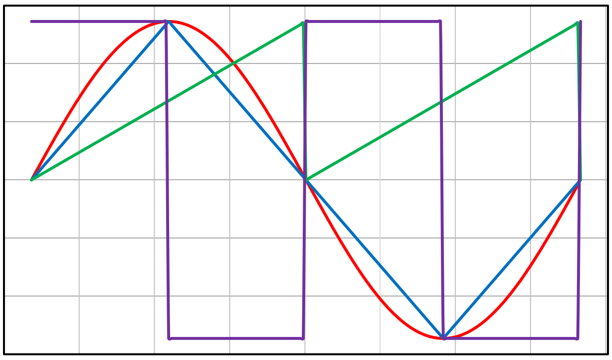

Figure 12: Qualitative depictions of waveforms.

These examples illustrate how the THD values, given in %, increases the further the waveform deviates from the sine shape. It is almost intuitive. In the given examples, the triangle is in its appearance very similar to the sine wave and has, apart from the pure sine wave, the smallest THD (Table 3). The saw tooth on the other hand shows the strongest deviation and has the largest THD.

Signal	THD, Equation	THD
saw tooth wave	$100\,\% \cdot \sqrt{\frac{\pi^2}{6} - 1}$	80.3%
square wave	$100\,\% \cdot \sqrt{\frac{\pi^2}{8} - 1}$	48.3%
triangle wave	$100\,\% \cdot \sqrt{\frac{\pi^4}{8} - 1}$	12.1%
sine wave	$100\,\% \cdot \sqrt{\frac{0}{1}}$	0%

Table 3: Examples of commonly used signals and its THD.

Figure 13 shows a Fourier transformed signal of saw tooth wave of 20 harmonic components at a fundamental frequency of 500 Hz. Since the transformed signal only contains 20 components the THD is about 77% with more components it would approach the theoretical value of 80.3% as given in Table 3.

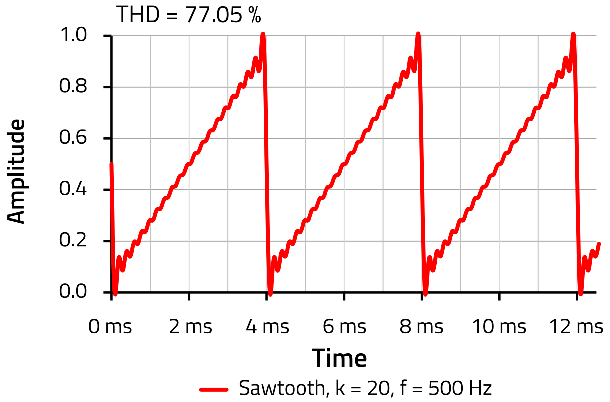

Figure 13: Example of a sawtooth signal with fundamental frequency of 500Hz and 20 higher harmonics.

B. Appendix, Capacitor Model

Appling Kirchhoff's rule to the circuit in Figure 3 leads to

$$V_s = V_R + V_L + V_C \tag{6}$$

with V_s as the time dependent voltage signal of applied by the source, V_R as the voltage drop over the ESR and V_L as the voltage drop over the ESL.

The substitution of the voltages with

$$V_R = IR = RC\frac{dV_C}{dt} \tag{7}$$

and

$$V_L = L\frac{dI}{dt} = LC\frac{d^2}{dt^2} \tag{8}$$

yields

$$V_s = RC\frac{dV_c}{dt} - LC\frac{d^2V_c}{dt^2} + V_C \tag{9}$$

The solution of equation (9) with the Ansatz

$$V_s(t) = V_0 \operatorname{Cos}(\omega t) \tag{10}$$

and

$$V_C(t) = A\operatorname{Cos}(\omega t + \phi) \tag{11}$$

were V_0 is the signal amplitude, ω is the signal frequency, A is a voltage amplitude and ϕ is the phase shift of the capacitor response, yields

$$I(t) = \frac{C\omega V_0}{(LC\omega^2 + 1)\cos(\phi) - RC\sin(\phi)}\cos\left(\omega t + \phi + \frac{\pi}{2}\right) \tag{12}$$

$$V_C(t) = \frac{V_0}{(LC\omega^2 + 1)\cos(\phi) - RC\sin(\phi)}\cos(\omega t + \phi) \tag{13}$$

$$V_L(t) = -\frac{LC\omega^2 V_0}{(LC\omega^2 + 1)\cos(\phi) - RC\sin(\phi)}\sin\left(\omega t + \phi + \frac{\pi}{2}\right) \tag{14}$$

as well as

$$\phi = \arctan\left(-\frac{RC\omega}{LC\omega^2 + 1}\right) \tag{15}$$

The equations (12), (13), (14) and (15) allow for a frequency shift, however, they do not contain oscillating terms with other frequencies than the signal frequency ω. Since, the capacitor is not causing any harmonics the THD is zero.

To estimate the magnitude of the THD of a nonlinear capacitance the constant term C is replaced by a voltage dependent capacitance

$$\begin{aligned} C(V,t) &= C_0\left(1 - aV_s(t)\right) \\ &= C_0(1 - aV_0 \operatorname{Cos}(\omega t)) \end{aligned} \tag{16}$$

were C_0 is the capacitance at zero volts and a a factor that governs the voltage dependence. By doing so, it is assumed that the capacitance is changed instantaneously upon the application of an external field, i.e. the dependence of the capacitance is proportional to $V_s(t)$.

The DC voltage dependence for an electrolytic capacitor above the nominal voltage is on average never greater than 1%, as shown in Table 4, Table 5 and Table 6 for various types of electrolytic capacitors.

C. Appendix, Formulas, Tables and Notes

To calculate sound pressure attenuation L_2 over distance for a point sound in an anechoic environment:

$$L_2(r_2) = L_1(r_1) - 20 \, Log_{10}\left(\frac{r_2}{r_1}\right) \qquad (17)$$

Where $L_1(r_1)$ is the known sound pressure level at distance r_1 and $L_2(r_2)$ is the unknown sound pressure level at the second distance r_1 [7]

Example: Trumpet [14]

Variable [Unit]	Value
$L_1(r_1)$ [dB]	120
r_1 [m]	1
$L_2(r_2)$ [dB]	40
r_2 [m]	10000

DC Voltage [% of V_r]	C [nF]	ΔC [%]
0	90.256	0.000
2	90.198	-0.064
4	90.166	-0.099
6	90.138	-0.130
8	90.117	-0.154
10	90.095	-0.177
20	90.071	-0.204
30	90.076	-0.198
40	90.109	-0.161
50	90.161	-0.105
60	90.226	-0.033
70	90.301	0.051
80	90.383	0.142
90	90.470	0.238
100	90.559	0.337

Table 4: Measurements taken at 120 Hz of: Series: WCAP-ATG8, 860010672001, 100 nF, AC-Level: 1 V, Keysight E4980A, Fixture: 16065A.

DC Voltage [% of V_r]	C [nF]	ΔC [%]
0	107.346	0.000
2	107.281	-0.061
4	107.261	-0.080
6	107.218	-0.122
8	107.154	-0.181
10	107.095	-0.237
20	107.032	-0.296
30	107.044	-0.284
40	107.095	-0.238
50	107.171	-0.167
60	107.267	-0.078
70	107.378	0.026
80	107.504	0.143
90	107.643	0.273
100	107.796	0.415

Table 5: Measurements taken at 1 kHz of: Series: WCAP-ASLU, 865090640001, 100 nF, AC-Level: 1 V, Keysight E4980A, Fixture: 16065A.

DC Voltage [% of V_r]	C [µF]	ΔC [%]
0	465.552	0.000
2	465.117	-0.093
4	464.741	-0.174
6	464.436	-0.240
8	464.174	-0.296
10	463.910	-0.353
20	463.070	-0.533
30	462.433	-0.670
40	462.456	-0.665
50	462.810	-0.589
60	463.682	-0.402
70	464.843	-0.152
80	466.137	0.143
90	467.913	0.507
100	469.796	0.912

Table 6: Measurements taken at 120 Hz of: Series: WCAP-ASLI, 865080253012, 470 µF, AC-Level: 0.2 V, impedance analyzer Alpha-AK, POT/GAL from Novocontrol in a four-terminal Kelvin configuration.

The harmonics, given in Table 7 and Table 8, are at the auditory sensation threshold and have been determined in a psychoacoustic experiment by Hong Yue Lin. [13] In this experiment the test listener heard the fundamental frequency at 90 dB. Each individual harmonic could be superimposed individually. The listener then adjusted the volume of the harmonic frequency to the hearing threshold.

The corresponding THD for those harmonics are about 10%. The THD of the first 10 harmonics are 9.4% and 7.3% for fundamentals of 50 Hz and 500 Hz, respectively. In correspondence to the auditory sensitivity, the amplitudes are smallest at 5 kHz and increase toward low and high frequencies. The values in the Table 8 are inversely proportional to the trend of the ITU-R 468 filter curve in Figure 2 in section 02 and, thus, reflect the sensitivity of the human ear.

The comparison of the percentage decrease of amplitudes in Table 7 and Figure 6 show that the harmonics of the capacitor (fundamental frequency about 500 Hz) are well below the auditory perception. The same result is found when comparing the amplitudes measured at fundamental about 50 Hz, given in Table 8 and Figure 14.

No.	Freq. [Hz]	diff. Values [dB]	Value [dB]	Values [%]
1	500	90	90	100.000
2	1000	-23.7	66.3	6.531
3	1500	-30.8	59.2	2.884
4	2000	-36.3	53.7	1.531
5	2500	-43.7	46.3	0.653
6	3000	-50.8	39.2	0.288
7	3500	-57.9	32.1	0.127
8	4000	-57.1	32.9	0.140
9	4500	-60.75	29.25	0.092
10	5000	-64.4	25.6	0.060
11	5500	-60.35	29.65	0.096
14	7000	-56.3	33.7	0.153
18	9000	-59.8	30.2	0.102
24	12000	-56.3	33.7	0.153
32	16000	-24.7	65.3	5.821

Table 7: The characteristics of auditory sensation threshold at various harmonics of a fundamental frequencies = 500 Hz. Each signal amplitude above the 1st fundamental is at the auditory sensation threshold. [13] The values for the 6th, 8th, 9th and 11th harmonic have been obtained by interpolated and used for the THD calculation.

No.	Freq. [Hz]	diff. Values [dB]	Value [dB]	Values [%]
1	50	90	90	100.00
2	100	-21.2	68.8	8.7096
3	150	-30.5	59.5	2.9854
4	200	-36.2	53.8	1.5488
5	250	-40.8	49.2	0.9120
6	300	-46.2	43.8	0.4898
7	350	-51.6	38.4	0.2630
8	400	-57.65	32.35	0.1311
9	450	-58.225	31.775	0.1227
10	500	-58.8	31.2	0.1148
11	550	-61.25	28.75	0.0866
14	700	-63.7	26.3	0.0653
20	1000	-67.9	22.1	0.0403
28	1400	-71.5	18.5	0.0266
60	3000	-77.2	12.8	0.0138
80	4000	-77.8	12.2	0.0129
100	5000	-76	14	0.0158
132	6600	-73	17	0.0224
240	12000	-54.9	35.1	0.1799
320	16000	-25.6	64.4	5.2481

Table 8: The characteristics of auditory sensation threshold at various harmonics of a fundamental frequency = 50 Hz. Each signal amplitude above the 1st fundamental is at the auditory sensation threshold.[13] The values for the 6th, 8th, 9th and 11th harmonic have been obtained by interpolated and used for the THD calculation.

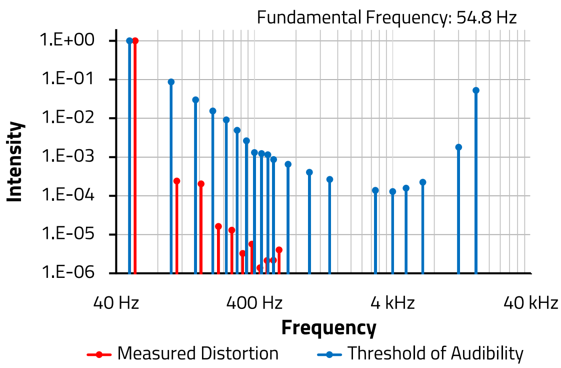

Figure 14: Measured frequency spectrum of 470 µF aluminum electrolytic capacitor (Capxon, P4, 317000012224000) at fundamental frequency of 54.8 Hz (ac voltage signal). Also shown, threshold of audible distortions as determined in psychoacoustic experiment for a fundamental at 500 Hz. [13]

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